

# NMHDECAY: A Fortran Code for the Higgs Masses, Couplings and Decay Widths in the NMSSM

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## Abstract

The Fortran code NMHDECAY computes the masses, couplings and decay widths of all Higgs bosons of the NMSSM in terms of its parameters at the electroweak (or SUSY breaking) scale: the Yukawa couplings  $\lambda$  and  $\kappa$ , the soft trilinear terms  $A_\lambda$  and  $A_\kappa$ , and  $\tan\beta$  and  $\mu_{\text{eff}} = \lambda \langle S \rangle$ . The computation of the spectrum includes leading two loop terms, electroweak corrections and propagator corrections. The computation of the decay widths is carried out as in HDECAY, but (for the moment) without three body decays. Each point in parameter space is checked against negative Higgs bosons searches at LEP, including unconventional channels relevant for the NMSSM. One version of the program uses generalized SLHA conventions for input and output.

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# 1 Introduction

The Next to Minimal Supersymmetric Standard Model (NMSSM [1–4, 6–12]) provides a very elegant solution to the  $\mu$  problem of the MSSM via the introduction of a singlet superfield  $\widehat{S}$ . For the simplest possible scale invariant form of the superpotential, the scalar component of  $\widehat{S}$  acquires naturally a vacuum expectation value of the order of the SUSY breaking scale, giving rise to a value of  $\mu$  of order the electroweak scale. The NMSSM is actually the simplest supersymmetric extension of the standard model in which the electroweak scale originates from the SUSY breaking scale only.

In addition, the NMSSM renders the “little fine tuning problem” of the MSSM, originating from the non-observation of a neutral CP-even Higgs boson at LEP II, less severe [2].

A possible cosmological domain wall problem [3] can be avoided by introducing suitable non-renormalizable operators [4] that do not generate dangerously large singlet tadpole diagrams [5].

Hence, the phenomenology of the NMSSM deserves to be studied at least as fully and precisely as that of the MSSM. Its particle content differs from the MSSM by the addition of one CP-even and one CP-odd state in the neutral Higgs sector (assuming CP conservation), and one additional neutralino. Thus, the physics of the Higgs bosons – masses, couplings and branching ratios [1, 6–12] can differ significantly from the MSSM. The purpose of the Fortran code NMHDECAY (Non Minimal Higgs Decays), that accompanies the present paper, is an accurate computation of these properties of the Higgs bosons in the NMSSM in terms of the parameters in the Lagrangian. As its name suggests, the Fortran code uses to some extent – for MSSM-like processes – parts of the code HDECAY that is applicable to the Higgs sector of the MSSM [13].

In the present paper we define the NMSSM in terms of its parameters at the SUSY breaking scale. No assumption on the soft terms (like universal soft terms at a GUT scale) are made. The parameters in the Higgs sector are chosen as follows:

- a) Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential is

$$\lambda \widehat{S} \widehat{H}_u \widehat{H}_d + \frac{\kappa}{3} \widehat{S}^3 \quad (1.1)$$

depending on two dimensionless couplings  $\lambda$ ,  $\kappa$  beyond the MSSM. (Hatted

capital letters denote superfields, and unhatted capital letters will denote their scalar components).

b) The associated trilinear soft terms are

$$\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3. \quad (1.2)$$

c) The final two input parameters are

$$\tan \beta = \langle H_u \rangle / \langle H_d \rangle, \quad \mu_{\text{eff}} = \lambda \langle S \rangle. \quad (1.3)$$

These, along with  $M_Z$ , can be viewed as determining the three SUSY breaking masses squared for  $H_u$ ,  $H_d$  and  $S$  through the three minimization equations of the scalar potential.

Thus, as compared to two independent parameters in the Higgs sector of the MSSM (often chosen as  $\tan \beta$  and  $M_A$ ), the Higgs sector of the NMSSM is described by the six parameters

$$\lambda, \kappa, A_\lambda, A_\kappa, \tan \beta, \mu_{\text{eff}}. \quad (1.4)$$

We will choose sign conventions for the fields such that  $\lambda$  and  $\tan \beta$  are positive, while  $\kappa$ ,  $A_\lambda$ ,  $A_\kappa$  and  $\mu_{\text{eff}}$  should be allowed to have either sign. For any choice of these parameters – as well as of the values for the gaugino masses and of the soft terms related to the squarks and sleptons that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths – NMHDECAY performs the following tasks:

1. It computes the masses and couplings of all physical states in the Higgs, chargino and neutralino sectors. Error messages are produced if a Higgs or squark mass squared is negative.
2. It computes the branching ratios into two particle final states (including charginos and neutralinos — decays to squarks and sleptons will be implemented in a later release) of all Higgs particles.
3. It checks whether the Higgs masses and couplings violate any bounds from negative Higgs searches at LEP, including many quite unconventional channels that are relevant for the NMSSM Higgs sector. It also checks the bound on the invisible  $Z$  width (possibly violated for light neutralinos). In addition,

NMHDECAY checks the bounds on the lightest chargino and on neutralino pair production. Corresponding warnings are produced in case any of these phenomenological constraints are violated.

4. It checks whether the running Yukawa couplings encounter a Landau singularity below the GUT scale. A warning is produced if this happens.
5. Finally, NMHDECAY checks whether the physical minimum (with all vevs non-zero) of the scalar potential is deeper than the local unphysical minima with vanishing  $\langle H_u \rangle$  or  $\langle H_d \rangle$ . If this is not the case, a warning is produced.

The web sites

<http://www.th.u-psud.fr/NMHDECAY/nmhdecay.html>

<http://higgs.ucdavis.edu/nmhdecay/nmhdecay.html>

provide links for downloading two different Fortran codes, along with descriptions, input files, and the data files that are required to apply the various LEP constraints.

The Fortran code NMHDECAY\_SLHA.f reads from input files, and produces output files, that are as close as possible to the SUSY Les Houches Accord (SLHA) conventions [14]. Some generalizations of these conventions – including proposals for PDG numbers – have been necessary, however, in order to denote the NMSSM input parameters (1.4) and the additional particles in the Higgs and neutralinos sectors.

The Fortran code NMHDECAY\_SCAN.f reads from “private” input files (samples are provided) that are constructed so as to scan over the NMSSM input parameters (1.4). Here the output is either “long” (easily human readable, if the number of points in parameter space is not too large), or “short”, i.e. simple rows of numbers per point in parameter space, that should be edited according to the user’s needs.

Note that the “long” output also gives the reduced couplings of all neutral Higgs bosons to gauge bosons (CV), up type quarks (CU), down type quarks (CD), two gluons (CG) and two photons (CGA) (all relative to a Standard Model Higgs boson of the same mass). Using these, it is easy to compute the NMSSM Higgs production cross sections at colliders by rescaling those for the SM Higgs boson.

The outline of the paper is as follows: In section 2, we describe in detail the accuracy with which the Higgs masses and mixing matrices are computed. In section 3, we describe the Higgs decay channels and corresponding accuracies that are used for the computation of the widths and branching ratios. In section 4, we describe the various phenomenological constraints that can be applied to the model.

Section 5 contains a user’s guide for the Fortran codes, as well as our generalizations of the SLHA conventions. In section 6, we show some results for branching ratios produced with the present code, emphasizing decay channels that are atypical of the MSSM, such as the lightest CP-even state  $h_1$  decaying to 2 lighter CP-odd states  $a_1a_1$ . Section 7 contains our conclusions. Appendix A contains a summary of our conventions, appendix B a summary of the Feynman rules, and appendix C the details of the radiative corrections to the higgs mass matrices.

## 2 Radiative Corrections in the Higgs Sector

Our convention for the superpotential, the soft terms, the resulting tree level potential and tree level mass matrices are given in appendix A. In the present section, we describe the accuracy with which radiative corrections are computed for the Higgs sector.

First, we assume that the Yukawa couplings and soft terms are defined at a SUSY breaking scale  $Q = M_{\text{SUSY}}$ , which corresponds to an average of the squark masses. Quantum fluctuations at scales  $> Q$  are assumed to be integrated out, which corresponds to the standard RG evolution of the parameters from a fundamental scale like  $M_{\text{GUT}}$  down to  $M_{\text{SUSY}}$ . The effective Lagrangian at the scale  $Q$  can be assumed to be of the standard supersymmetric form plus soft terms.

We are interested in the full effective action

$$\Gamma_{\text{eff}} = \sum_i Z_i D_\mu H_i D^\mu H_i - V_{\text{eff}}(H_i) , \quad (2.1)$$

that is obtained from the effective Lagrangian at the scale  $Q$  by adding all quantum effects arising from fluctuations at scales  $\lesssim Q$ . (Here,  $H_i$  denotes all the Higgs fields,  $H_u$ ,  $H_d$  and  $S$ .) These quantum effects can be classified according to powers of the various couplings, and powers of potentially large logarithms. Explicit formulas for the radiative corrections to  $V_{\text{eff}}$ ,  $Z_i$  and hence to the Higgs mass matrices are given in appendix C, subsequently we describe only the orders to which radiative corrections are taken into account.

Let us start with the corrections to the effective potential. It is somewhat more convenient, however, to classify the corrections to the (lightest) scalar Higgs mass, which is essentially the second derivative of the effective potential. At tree level this

mass squared is bounded by

$$m_1^2 < M_Z^2 \left( \cos^2 2\beta + \frac{2\lambda^2 \sin^2 2\beta}{g_1^2 + g_2^2} \right) , \quad (2.2)$$

which is of the order

$$m_1^2 \sim \mathcal{O}(g^2 v^2) + \mathcal{O}(\lambda^2 v^2) \quad (2.3)$$

where  $g$  denotes the electroweak gauge couplings, and  $v$  the magnitude of the Higgs vacuum expectation values (we do not distinguish between large and small  $\tan \beta$  in this section).

The dominant one-loop corrections to  $m_1^2$  originate from top, stop, bottom and sbottom loops. The corresponding corrections  $\delta m_1^2$  to  $m_1^2$  are of order

$$\begin{aligned} \delta_{\text{top}} m_1^2 &\sim h_t^4 v^2 \ln(Q^2/m_{\text{top}}^2) , \\ \delta_{\text{stop}} m_1^2 &\sim h_t^4 v^2 \quad (\text{for } Q^2 \sim m_{\text{stop}}^2) , \\ \delta_{\text{bottom}} m_1^2 &\sim h_b^4 v^2 \ln(Q^2/m_{\text{bottom}}^2) , \\ \delta_{\text{sbottom}} m_1^2 &\sim h_b^4 v^2 \quad (\text{for } Q^2 \sim m_{\text{sbottom}}^2) . \end{aligned} \quad (2.4)$$

We have included these contributions [6, 7, 11] exactly, without expanding in large logarithms or squark mass splittings.

The dominant two loop corrections are of the form [11, 12]

$$\delta^{(2)} m_1^2 \sim h_t^6 v^2 \ln^2(Q^2/m_{\text{top}}^2) + h_t^4 \alpha_s v^2 \ln^2(Q^2/m_{\text{top}}^2) . \quad (2.5)$$

Here, we only include the leading double logarithms, i.e. we neglect single logs as well as terms involving  $h_b^2$ .

Coming back to the one-loop order, we focus on including all contributions of the order  $g^4 \times$  large logarithms. These can be obtained by RG methods [15], taking care of the fact that the electroweak couplings  $g$  are defined at the scale  $M_Z$  and not at the scale  $M_{\text{SUSY}}$ . We have included all these contributions as given in [15], distinguishing large logarithms  $\ln(Q^2/M_Z^2)$  from  $\ln(M_A^2/M_Z^2)$  and  $\ln(\mu_i^2/M_Z^2)$  ( $\mu_i = M_1, M_2$  or  $\mu_{\text{eff}}$ , where  $M_i$  are the electroweak gaugino masses). We have not (yet) included terms of the order  $g^2 \lambda^2 \times$  large logarithms, nor terms of the order  $g^4 \times$  finite.

This concludes our contributions to the effective potential. Next, we outline the contributions to the wave function normalization constants  $Z_i$  in (2.1). If evaluated for external momenta of  $\mathcal{O}(m_{\text{top}})$  (the order of the Higgs masses), top and bottom quark loops yield contributions to  $\delta Z_i$  (relative to 1) of the form

$$\delta Z_1 \sim h_t^2 \ln(Q^2/m_{\text{top}}^2) , \quad \delta Z_2 \sim h_b^2 \ln(Q^2/m_{\text{top}}^2) . \quad (2.6)$$

After rescaling the Higgs fields so that their kinetic energies are canonically normalized, these effects generate contributions  $\delta_Z m_1^2$  which take the form

$$\delta_Z m_1^2 \sim g^2 h_{t/b}^2 v^2 \ln(Q^2/m_{\text{top}}^2) + \lambda^2 h_{t/b}^2 v^2 \ln(Q^2/m_{\text{top}}^2) . \quad (2.7)$$

However, after diagonalization of the Higgs mass matrices, one may find eigenstates with masses  $m_H$  substantially larger than  $m_{\text{top}}$ . Then the  $Z$  factors should be evaluated at external momenta of  $\mathcal{O}(m_H)$ , i.e. at the pole of the propagators. Hence for  $m_H \gg m_{\text{top}}$  the logarithms  $\ln(Q^2/m_{\text{top}}^2)$  in (2.6) should be replaced by  $\ln(Q^2/m_H^2)$ , with coefficients depending on Higgs mixing matrices.

All this is done in the program, with the net result that all potentially large logarithms with coefficients  $g^2 h_{t/b}^2$  or  $\lambda^2 h_{t/b}^2$  have the correct arguments. (We neglect, however, contributions of  $\mathcal{O}(\ln(m_H^2/m_{\text{top}}^2))$  in the off diagonal Higgs matrix elements, which affects only the couplings of very heavy Higgses.) Since we have neglected terms of  $\mathcal{O}(h_{t/b}^2 \times \text{finite})$  in the  $Z$  factors and electroweak corrections without large logarithms, the dominant sources of uncertainty in  $\delta m^2$  are contributions of  $\mathcal{O}(g^4 v^2 \times \text{finite})$ ,  $\mathcal{O}(g^2 h_{t/b}^2 v^2 \times \text{finite})$  and  $\mathcal{O}(\lambda^2 h_{t/b}^2 v^2 \times \text{finite})$  as well as subdominant (single log) two loop corrections.

We have checked that in the end, after expanding the  $Z$  factors appropriately, all  $Q^2$  dependence in the one-loop contributions can be reabsorbed into the running of the input parameters that are defined at the scale  $Q^2$ . An exception is the  $Q^2$  dependence of terms  $\sim g^4$ , since we have not included the squark loop contributions  $\sim g^4$  (from squark couplings to Higgs bosons arising from the  $D$ -terms). These would give contributions to  $m_1^2$  of the order

$$\delta_D m_1^2 \sim g^4 \ln(Q^2/m_{\text{squark}}^2) , \quad (2.8)$$

which is relatively small for  $Q^2 \sim m_{\text{squark}}^2$ .

Since we have not included contributions of the orders  $(g^4, g^2 h_{t,b}^2) \times \text{finite}$ , we are actually not sensitive to the scheme in which the electroweak couplings are defined. Since in the computation of the decay widths we use the physical masses  $M_W$  and  $M_Z$  in order to parameterize some couplings, the most useful scheme here is the on-shell scheme, where the three “unphysical” electroweak parameters  $v^2 = \langle H_u^2 + H_d^2 \rangle$ ,

$g_1^2$  and  $g_2^2$  are defined in terms of  $G_F$ ,  $M_W$  and  $M_Z$  as

$$\begin{aligned} v^2 &= \frac{1}{2\sqrt{2} G_F}, \\ g_2^2 &= 2M_W^2/v^2, \\ g_1^2 + g_2^2 &= 2M_Z^2/v^2. \end{aligned} \quad (2.9)$$

Finally we remark that we have applied the above corrections consistently to the complete Higgs sector, i.e. the CP-even, CP-odd and charged Higgs states. One finds that one can reabsorb several of the radiative corrections into a redefinition of the input parameter  $A_\lambda$  [6], after which the CP-odd mass matrix assumes its tree level form, up to the required rescalings of the vevs by the  $Z$  factors. This phenomenon is familiar from the MSSM, where  $M_A$  is thus a convenient input parameter, and the required rescalings are often absorbed into a scale dependent value of  $\tan \beta$ .

We have checked that in the MSSM limit of the NMSSM ( $\lambda, \kappa \ll 1$ ), after comparing for the same values of  $\tan \beta$  and  $M_A$  (and the remaining soft terms), the mass of the lightest CP-even state agrees with the one computed in HDECAY [13] (models 1 [16] or 2 [18]) to better than 2 GeV for moderate mixing ( $A_{\text{top}} \lesssim 1$  TeV), and to better than 3 GeV for maximal mixing. This coincides with the expected theoretical error on this mass given the orders of uncertainty discussed above.

A more detailed discussion of the radiative corrections to the NMSSM Higgs sector will appear in [17].

### 3 Higgs Decays

In this section we describe the decay modes that are included in the Fortran code. Most of the corresponding code is extracted from HDECAY [13]. In what follows,  $H$  denotes any of the 3 CP-even or 2 CP-odd scalars, or the charged Higgs boson.

a)  $H \rightarrow$  gluons: We take into account charm, bottom and top quark loops; the lowest order contribution to the decay width is given in [19]. QCD radiative corrections [20] are included at the same level as in HDECAY. Squark loops are not (yet) taken into account.

b)  $H \rightarrow$  leptons and quarks: QCD corrections are included as in HDECAY [21], but we use always the two loop formula for the relation between the pole quark mass  $m_{\text{pole}}$  and the running  $\overline{MS}$  quark mass  $\overline{m}(m_{\text{pole}})$ :

$$\overline{m}_i(m_{\text{pole}}) = \frac{m_{\text{pole}}}{1 + \frac{4\alpha_s(m_{\text{pole}})}{3\pi} + K_i \left( \frac{\alpha_s(m_{\text{pole}})}{\pi} \right)^2} \quad (3.1)$$

with  $K_{\text{bottom}} \sim 12.4$ ,  $K_{\text{top}} \sim 11$ . The scale dependence of the running  $s$  and  $c$  quark masses is computed as in HDECAY, but for the running  $b$  and  $t$  quark masses we use running Yukawa couplings that solve the one-loop RG equations exactly in  $\alpha_s$ , improved by corrections to first order in the Yukawa couplings themselves (see appendix C).

Since electroweak corrections to decays into leptons and quarks are small [13], they are neglected. Below-threshold three-body decays  $H \rightarrow t\bar{t} \rightarrow t\bar{b}W^-$  are not (yet) taken into account. However, contributions to the decay rates into  $b\bar{b}$  ( $c\bar{c}$ ) from  $H \rightarrow gg^* \rightarrow g\bar{b}b$  ( $c\bar{c}$ ) are included as in HDECAY (option NF-GG = 3, that can easily be modified by re-editing the line NFGG = 3 at the beginning of the subroutine DECA). SUSY loop corrections are not (yet) included. (These could be important for light squarks and/or large  $\tan\beta$ .) In the case of a decaying charged Higgs boson, we must input the CKM matrix elements  $V_{us}$ ,  $V_{ub}$  and  $V_{cb}$ .

- c)  $H \rightarrow WW/ZZ$ : We treat these decays as in HDECAY [22], but without the double off-shell option.
- d)  $H \rightarrow \gamma\gamma/Z\gamma$ : We take into account loops of charm, bottom and top quarks,  $\tau$  leptons,  $W$  bosons, charginos and charged Higgs bosons [23]. Loops of squarks and sleptons are not (yet) considered.
- e)  $H \rightarrow HH$ : All kinematically possible combinations ( $H = \text{CP-even, CP-odd or charged}$ ) are considered, with  $(\text{Higgs})^3$ -couplings computed from the NMSSM potential (see F. Franke and H. Fraas in ref. [1] and appendix B). The dominant (leading log) radiative corrections from top-quark and bottom-quark loops to these couplings are included in the Fortran code, the corresponding formulas will appear in [17]. Decays into three particle final states are not (yet) taken into account.
- f)  $H \rightarrow HZ/HW$ : All kinematically possible combinations ( $H = \text{CP-even, CP-odd or charged}$ ) are considered [24]. Below threshold decays into three particle final states are not (yet) taken into account.
- g)  $H \rightarrow \text{charginos/neutralinos}$ : First, the  $2 \times 2$  chargino and  $5 \times 5$  neutralino mass matrices are diagonalized in the subroutines CHARGINO and NEUTRALINO, and the decays into all possible two body final states [25] are included in the program.

Decays into sleptons or squarks are not (yet) considered. Note that relatively

light squarks – even if too heavy to be produced in Higgs decays – could nevertheless affect notably the processes b) and d) above through squark loops. For this reason the present version of NMHDECAY should not be applied to scenarios with light squarks.

## 4 Constraints on the Parameter Space

Once the spectrum and decay widths of the Higgses are computed, we check some experimental and theoretical constraints for each point in parameter space:

First, we require that there is no Landau pole for the Yukawa couplings  $\lambda, \kappa, h_{\text{top}}$  and  $h_{\text{bot}}$  below the GUT scale. For this, we integrate the two loop renormalization group equations for the Yukawa and gauge couplings from the SUSY scale up to the point where the gauge couplings  $g_1$  and  $g_2$  unify. At this point we check that none of the couplings is too large (i.e. exceeds  $\sqrt{4\pi}$ ). (For simplicity we assume a universal SUSY threshold at  $M_{\text{susy}} \sim Q$ .)

We also check that there is no deeper minimum of the scalar potential with vanishing  $\langle H_u \rangle$  or  $\langle H_d \rangle$ .

Finally we check all available experimental constraints from LEP:

- 1) In the neutralino sector, we check that the lightest neutralino does not contribute too much to the invisible  $Z$  width ( $\Gamma(Z \rightarrow \chi_1^0 \chi_1^0) < 1.76$  MeV [26]) if  $m_{\chi_1^0} < M_Z/2$ , and that  $\sigma(e^+e^- \rightarrow \chi_1^0 \chi_i^0) < 10^{-2}$  pb if  $m_{\chi_1^0} + m_{\chi_i^0} < 209$  GeV ( $i > 1$ ) and  $\sigma(e^+e^- \rightarrow \chi_i^0 \chi_j^0) < 10^{-1}$  pb if  $m_{\chi_i^0} + m_{\chi_j^0} < 209$  GeV ( $i, j > 1$ ) [27]. In the chargino sector, we check that the lightest chargino is not too light ( $m_{\chi_1^\pm} < 103.5$  GeV [28]).
- 2) In the charged Higgs sector, we check the bound  $m_{H^+} > 78.6$  GeV [29].
- 3) In the neutral Higgs sector, we check the constraints on the production rates (reduced couplings)  $\times$  branching ratios versus the masses, for all of the CP-even states  $h$  and CP-odd states  $a$ , in the following channels studied at LEP:

- $e^+e^- \rightarrow hZ$ , with  $h \rightarrow b\bar{b}$  and  $h \rightarrow \tau^+\tau^-$  (from the LEP Higgs working group results [30]);
- $e^+e^- \rightarrow hZ$ , with  $h$  decaying to two jets. For this, we combined the low mass range results from OPAL at LEP2 [31] with the higher mass range obtained at LEP2 by the LEP Higgs Working group [32];
- $e^+e^- \rightarrow hZ$ , with  $h \rightarrow \gamma\gamma$  (from the LEP Higgs working group results [33]);

- $e^+e^- \rightarrow hZ$ , with  $h$  decaying invisibly (*i.e.* into two neutralinos). For this, we combined the low mass range results from ALEPH at LEP1 [34] with the higher mass range obtained at LEP2 by the LEP Higgs Working group [35].
- $e^+e^- \rightarrow hZ$ , independent of the  $h$  decay mode, looking for a peak of the  $M_X$  recoil mass distribution in  $e^+e^- \rightarrow XZ$ . For this, we combined the low mass range results from ALEPH at LEP1 [34] with the higher mass range obtained at LEP2 by OPAL [36].
- For the associated production mode  $e^+e^- \rightarrow ha$  we used the DELPHI results [37] for the various final states:  $ha \rightarrow 4b$ ,  $ha \rightarrow 4\tau$  and  $ha \rightarrow aaa \rightarrow 6b$ . The latter decay channel plays an important role in the NMSSM, where the lightest CP-even Higgs can decay mainly into two light CP-odd Higgses over large areas of the parameter space, as we will see in section 6. (In the context of the CP-conserving MSSM, this is possible only for very special parameter choices.) DELPHI also studied the channel  $hZ \rightarrow aaZ \rightarrow 4b + 2\text{jets}$  [37]; their limits have also been implemented in our phenomenological constraints.
- The channels  $e^+e^- \rightarrow hZ \rightarrow aaZ$  with  $aa \rightarrow 4\text{jets}$ ,  $2\text{jets} + cc$ ,  $2\text{jets} + \tau\tau$ ,  $4\tau$ 's,  $cccc$ ,  $\tau\tau + cc$  have been studied by OPAL [38] (see their Fig. 7 — we employ numerical files for the contours provided by the authors). These limits have also been included.

All these constraints are contained in the files located in the directory LEPCON downloadable together with the Fortran codes. The file names are constructed so that they can be identified with the above delineated constraints. (The current numerical limits contained in these files can possibly be improved with the help of further dedicated analysis of existing data.)

## 5 How to use NMHDECAY

NMHDECAY exists in two versions:

1. NMHDECAY\_SLHA uses an input file and produces output files that are suitable generalizations of the SLHA conventions [14]. It is designed for studying the properties of one user-defined point in parameter space.

2. NMHDECAY\_SCAN uses input and output files that can be modified by the user so as to scan over parts of, or all of the NMSSM parameters defined in eq. (1.4).

Both programs have the common feature that they are based on one single Fortran code (NMHDECAY\_SLHA.f or NMHDECAY\_SCAN.f) that does not need to be linked with any other code. However, both programs need data files in order to check against negative Higgs searches at LEP in the numerous channels discussed in section 4. These data files are available in the directory LEPCON, that can be downloaded from the NMHDECAY home page. Note that the directory LEPCON has to be situated in the same directory that contains the executable NMHDECAY code. We now discuss the particular features of the two programs.

## 5.1 NMHDECAY\_SLHA

NMHDECAY\_SLHA uses an input file `slhainp.dat`, a version of which is downloaded automatically with the Fortran code. This sample file appears in Table 1. Several comments on its contents are in order.

- “BLOCK MODSEL” contains the entry 3 (corresponding to the choice of the particle content) with switch 1, as attributed to the NMSSM in [14].
- “BLOCK SMINPUTS” contains important Standard Model parameters.
  - First, there is the inverse electromagnetic coupling constant at the scale 0, which is that required for the computations of the decay widths into two (on-shell) photons.
  - Second, various Higgs couplings are defined in terms of  $M_Z$ ,  $\alpha_s(M_Z)$ ,  $M_W$  and  $G_F$ , which means an on shell scheme is implicitly being used in order to define the electroweak parameters (cf. the corresponding discussion in section 2).  $M_Z$ ,  $\alpha_s(M_Z)$  and  $G_F$  are input as part of “BLOCK SMINPUTS”, whereas the numerical value of  $M_W = 80.42$  GeV is defined in the subroutine INPUT, that reads also the input file `slhainp.dat`. (Part of the corresponding code is copied from `pyslha.f`, with thanks to P. Skands.)
  - Third, as part of this block we input the running  $b$  quark mass  $m_b(m_b)$ , the top quark pole mass and  $m_\tau$ .

4. In addition, NMHDECAY needs the (pole) quark masses  $m_s$  and  $m_c$ , as well as the CKM matrix elements  $V_{us}$ ,  $V_{cb}$  and  $V_{ub}$ . The numerical values of these five parameters are defined in the subroutine INPUT. (For convenience, they are printed out in the output file `spectr.dat`, see below.)

- c) “BLOCK MINPAR” contains only the switch 3 with the input value for  $\tan\beta$  specified.
- d) “BLOCK EXTPAR” contains inputs for the SUSY and soft-SUSY-breaking parameters. Needed generalizations of the SLHA conventions appear. The new entries are:

61	for $\lambda$
62	for $\kappa$
63	for $A_\lambda$
64	for $A_\kappa$

Note that the entry 23 ( $\mu$ ) is used for the effective  $\mu$  parameter,  $\mu_{\text{eff}} = \lambda \langle S \rangle$ , in the NMSSM. It should also be noted that neither the slepton entries (all generations) nor the squark entries for the first and second generations are actually used; these are included for future use only.

The two output files of NMHDECAY\_SLHA are `spectr.dat` (see Table 2) and `decay.dat`. In `spectr.dat`, “BLOCK SPINFO” is followed by warnings (switch 3) if any of the constraints described in section 4 are violated, or if any of the six Higgs states could decay into squarks in which case the corresponding widths and branching ratios should not be used. This segment of the output also displays error messages (switch 4) if any of the Higgs, stop or sbottom states have a negative mass squared. No spectrum output is produced in this case.

In the output, “BLOCK SMINPUTS” is followed by a printout of the Standard Model input parameters. The numerical values for  $M_W$ ,  $m_s$ ,  $m_c$ ,  $V_{us}$ ,  $V_{cb}$  and  $V_{ub}$ , that have no SLHA numbers, appear in lines subsequent to “# SMINPUTS Beyond SLHA”. Similarly, “BLOCK MINPAR” is followed by a printout of the value of  $\tan\beta$  and “BLOCK EXTPAR” is followed by a printout of the important SUSY and soft-SUSY-breaking parameters.

The (pole) masses for the Higgs particles, the neutralinos, the charginos and squarks appear in the output following “BLOCK MASS”. There, one finds several essential NMSSM generalizations of the SLHA conventions. The new entries, with proposed PDG codes, are

45	for the third CP-even Higgs boson,
46	for the second CP-odd Higgs boson,
1000045	for the fifth neutralino.

The Higgs mixings in the CP-even sector follow “BLOCK HIGMIX” and those in the CP-odd sector follow “BLOCK AMIX”. Both segments of the output contain NMSSM generalizations of the SLHA conventions, that are required in order to parameterize the mixing in the enlarged Higgs sector. The meaning of the matrix elements  $S_{ij}$  ( $i, j = 1, 2, 3$ ) and  $P'_{ij}$  ( $i, j = 1, 2$ ) is as follows.

- According to the SLHA conventions the Higgs states  $H_u, H_d$  are denoted by  $H_2, H_1$ , respectively. (Inside the Fortran code the Higgs states  $H_u, H_d$  are denoted by  $H_1, H_2$ , which is of no relevance for the SLHA output.) Hence, for the purpose of the SLHA output, the CP-even Higgs weak eigenstates are numbered by  $S_i^{weak} = (H_{dR}, H_{uR}, S_R)$  ( $R$  refers to the real component of the field). If  $h_i$  are the mass eigenstates (ordered in mass), the convention is  $h_i = S_{ij}S_j^{weak}$ .
- In the CP-odd sector the bare Higgs fields are  $H_{uI}, H_{dI}, S_I$  ( $I$  for imaginary component). Again, for the purpose of the SLHA output, the CP-odd Higgs weak eigenstates are denoted by  $H_{uI} = H_{2I}, H_{dI} = H_{1I}$ . The mass eigenstates are  $a_i$  (ordered in mass,  $i = 1, 2$ ) and the Goldstone mode  $\tilde{G}$ . Then the elements of  $P'_{ij}$  are defined as

$$\begin{aligned} H_{1I} &= \sin \beta (P'_{11}a_1 + P'_{21}a_2) + \cos \beta \tilde{G} , \\ H_{2I} &= \cos \beta (P'_{11}a_1 + P'_{21}a_2) - \sin \beta \tilde{G} , \\ S_I &= P'_{12}a_1 + P'_{22}a_2 . \end{aligned} \quad (5.1)$$

- In the output, “BLOCK NMIX” is followed by a printout of the obvious generalization of the  $4 \times 4$  MSSM neutralino mixing matrix to the  $5 \times 5$  NMSSM neutralino mixing matrix (with real entries); “BLOCK UMIX” and “BLOCK VMIX” are followed by printouts of the  $U$  and  $V$  matrices as defined in the MSSM.
- The output file `decay.dat` respects the SLHA conventions for the Decay file (at present we consider two particle final states only), using the above generalizations of the PDG codes both for the decaying particle and the final states.

## 5.2 NMHDECAY\_SCAN

NMHDECAY\_SCAN uses the input files `smpar.dat` and `scaninp.dat`, versions of which are downloaded automatically with the Fortran code. One of the sample files appears in Table 3.

In the file `smpar.dat` the following standard model parameters must be specified:  $\alpha_s(M_Z)$ ,  $G_F$ ,  $\alpha_{\text{e.m.}}^{-1}(0)$ , the lepton masses  $m_\tau$  and  $m_\mu$ ,  $M_Z$ ,  $M_W$ , the pole quark masses  $m_s$ ,  $m_c$ , the running bottom quark mass  $m_b(m_b)$ , the top quark pole mass  $m_t$ , and the CKM matrix elements  $V_{us}$ ,  $V_{cb}$  and  $V_{ub}$ .

In the file `scaninp.dat`, the following must be specified:

- the total number of points to be scanned in parameter space;
- the output format (0 for “short”, corresponding to simple rows of numbers per allowed point in parameter space, and 1 for “long”, as described below);
- lower and upper limits for the NMSSM parameters  $\lambda$ ,  $\kappa$ ,  $\tan\beta$ ,  $\mu_{\text{eff}}$ ,  $A_\lambda$  and  $A_\kappa$ ;
- the soft squark masses, trilinear couplings and gaugino masses over which no scan is performed — the slepton masses as well as the squark masses of the first two generations (without index 3) are not (yet) used.

The scan in parameter space uses a random number generator, such that all NMSSM parameters are randomly chosen point by point in the parameter space within the specified limits.

The output file containing the physical parameters is always called `scanout.dat`, regardless of the output format chosen. The numbers printed out for the output format 0 (recommended for scans over more than 10 points in parameter space) should be edited according to the user’s needs.<sup>¶</sup> The output format 1 is easily readable and shows

- the NMSSM parameters for each point,
- possible warnings in case any phenomenological constraint is violated, a strong coupling regime below  $M_{\text{GUT}}$  appears or an unphysical minimum is deeper than the physical one (cf. section 4), if any of the six Higgs states could decay into squarks (in which case the corresponding widths and branching ratios should not be used)

---

<sup>¶</sup>See the section of the program following the comment line “ The following 3 lines should be edited according to the user’s needs”.

or error messages ("fatal" errors) in case any of the Higgs, stop or sbottom states has a negative mass squared (in which case no additional output is produced).

- for each of the six Higgs states, their mass, their decomposition into weak eigenstates ( $H_u$ ,  $H_d$ ,  $S$ ) (in **Components**), their reduced couplings to gauge bosons (CV), up type quarks (CU), down type quarks (CD), two gluons (CG) and two photons (CGA) (all relative to a standard model Higgs boson with the same mass), their branching ratios (where "Higgses" denote all possible two Higgs final states, and "sparticles" all possible two particle neutralino/chargino final states), and their total width,
- the neutralino and chargino masses (all masses in GeV), as well as the neutralino composition in the basis  $\psi^0 = (-i\lambda_1, -i\lambda_2, \psi_u^0, \psi_d^0, \psi_s)$  described in detail in Appendix A.
- the (pole) stop and sbottom masses (in GeV).

The output file **scanerr.dat** shows how many of the points in parameter space have avoided fatal errors or violations of phenomenological constraints, and the range in the NMSSM parameter space over which points have passed all these tests.

For users who wish to call a subroutine as a function of the Higgs, chargino, neutralino and squark sector outputs, including mixing angles and other parameters and quantities computed during the course of the scan, they should use the parameters and common blocks found in **NMHDECAY\_SCAN.f** within "SUBROUTINE OUTPUT". The comments included in this subroutine should allow easy identification of all the parameters, branching ratios, mixing angles and so forth that would be of potential interest for inputting into a user's subroutine.

## 6 Results and discussion

As stated above, the masses, couplings and decay properties of the Higgs bosons of the NMSSM can differ significantly from the MSSM. The primary purpose of **NMHDECAY** is to allow for detailed studies of such cases.

A particularly interesting case is where the lightest Higgs scalar  $h_1$ , although primarily non-singlet, decays mainly into two pseudoscalars (also primarily non-singlet) [9]. It is then interesting to see how the branching ratios of  $h_1$  depend on the parameters of the model. In particular, we can determine whether the choices of parameters that yield the above types of decays are ruled out for some of the

phenomenological reasons discussed in section 4. We can also determine if there are particular (fine-tuned) relations between the parameters required for  $h_1 \rightarrow a_1 a_1$  to be dominant. To study such issues, it will be convenient to fix all but one of the parameters (which allows for a reasonable graphical representation), and perform a scan over the remaining parameter.

In some sense, the input parameter  $A_\lambda$  is the most natural one to vary, since the mass of the MSSM like pseudoscalar depends quite strongly on  $A_\lambda$  (and hence  $A_\lambda$  plays the role of  $M_A$  in the MSSM).

Let us first consider the following choice of the NMSSM parameters [cf. eq. (1.4)]:  $\lambda = \kappa = 0.3$ ,  $\tan \beta = 5$ ,  $\mu_{\text{eff}} = 180$  GeV,  $A_\kappa = 0$ . For the squark masses and trilinear couplings, we take 1 TeV and 1.5 TeV, respectively. Varying  $A_\lambda$  between 0 and 1000 GeV, we obtain the branching ratios for  $h_1$  as shown in fig. 1. These show clearly that, for  $A_\lambda \lesssim 600$  GeV, the decay  $h_1 \rightarrow a_1 a_1$  is dominant.

The reason for the sharp drop of the  $h_1 \rightarrow a_1 a_1$  branching ratio for  $A_\lambda > 600$  GeV becomes clear from fig. 2, where we show the masses  $m_{h_1}$  and  $m_{a_1}$  as functions of  $A_\lambda$ ; for  $A_\lambda > 600$  GeV,  $m_{a_1}$  becomes larger than  $m_{h_1}/2$ . The  $h_1 \rightarrow a_1 a_1$  decay is also reduced as  $A_\lambda \rightarrow 0$ , even though the  $a_1$  becomes very light, because in this case it is mainly singlet and has a very small coupling to the  $h_1$ . None of the points in these two graphs are excluded by LEP. They should be visible at the LHC using the techniques we have developed [9] for isolating the  $WW \rightarrow h \rightarrow aa$  type of signal.

For a second sample set of plots, figs. 3–6, we take  $\lambda = 0.5$ ,  $\kappa = -0.15$ ,  $\tan \beta = 3.5$ ,  $\mu_{\text{eff}} = 200$  GeV,  $A_\lambda = 780$  GeV and  $A_\kappa \in [150 \text{ GeV}, 250 \text{ GeV}]$ . The `scaninp.dat` file for this case is given in Table 3. For much of this parameter range, neither the  $h_1$  nor the  $h_2$  would have been observable at LEP. In particular, fig. 5 shows that  $m_{h_2} \gtrsim 120$  GeV implying that the  $h_2$  is beyond the LEP kinematical reach. The  $h_1$  is much lighter. However, this light Higgs is not excluded by LEP over most of the above  $A_\kappa$  range since: a) its reduced coupling to gauge bosons is small; and b)  $h_1 \rightarrow b\bar{b}$  is suppressed so that  $h_1 \rightarrow jj$  decays are dominant (see fig. 3). In fig. 6, we plot  $\xi^2 = C_V(h_1)^2 \times BR(h_1 \rightarrow jj)$  for our selected points as well as the region excluded by LEP searches in this channel [32]. We see that only if  $m_{h_1} \lesssim 53$  GeV, which corresponds to  $A_\kappa \gtrsim 235$  GeV, would the  $h_1$  be excluded by LEP data.

Will these Higgs bosons be observable at the LHC? In this regard, it is important

to note from fig. 4 that when  $A_\kappa \gtrsim 215$  GeV,  $h_2 \rightarrow h_1 h_1$  decays are dominant. This occurs because  $m_{h_1}$  decreases with  $A_\kappa$ , see fig. 5. Meanwhile, fig. 3 shows that  $BR(h_1 \rightarrow b\bar{b})$  and  $BR(h_1 \rightarrow \tau^+ \tau^-)$  are both small when  $A_\kappa \in [205 \text{ GeV}, 220 \text{ GeV}]$ ; in this region of parameter space, the  $h_1$  decays mainly to  $c\bar{c}$  or  $gg$ . Thus, for  $A_\kappa \sim 215 - 220$  GeV:

- The  $h_1$  has a mass that lies below the mass range currently studied for Higgs detection at the LHC. Further, the  $h_1$  will be so weakly produced at the LHC (since  $\xi^2 \lesssim 0.1$ ) that extensions to lower Higgs masses of the current LHC studies would probably conclude it was undetectable.
- Simultaneously, the strongly produced  $h_2$  has decays dominated by  $h_2 \rightarrow h_1 h_1$  with  $h_1 \rightarrow c\bar{c}, gg$  (but not  $b\bar{b}$  or  $\tau^+ \tau^-$ ). As a result, the techniques of [9] for  $h \rightarrow aa$  (which require a significant  $a \rightarrow \tau^+ \tau^-$  branching ratio) do not apply, and the  $h_2$  would also appear to be very difficult to observe at the LHC.

We note that the above choice of parameters produces a phenomenology for the Higgses that is somewhat similar to that discussed in [10], which focuses on a region of parameter space such that the  $h_1$  has suppressed decays to  $b\bar{b}$  and  $\tau^+ \tau^-$ . The difference is that the decay of their  $h_2$  to  $h_1 h_1$  is kinematically forbidden and the  $h_2$  could be detected at the LHC.

## 7 Conclusions

In this paper, we have presented details regarding the now publicly available NMHDECAY programs, NMHDECAY\\_SLHA.f and NMHDECAY\\_SCAN.f, that can be used to explore the Higgs sector of the Next to Minimal Supersymmetric Model defined by adding one singlet Higgs superfield to the Minimal Supersymmetric Model. The programs, and associated data files, can be downloaded from the two web pages:

<http://www.th.u-psud.fr/NMHDECAY/nmhdecay.html>  
<http://higgs.ucdavis.edu/nmhdecay/nmhdecay.html>

The web pages provide simplified descriptions of the programs and instructions on how to use them. The programs will be updated to include additional features and refinements in subsequent versions. We welcome comments with regard to improvements that users would find helpful.

We have exemplified the use of these programs for two particularly interesting scenarios: The first illustrates the potentially crucial importance of the LHC  $h \rightarrow aa$  detection mode (that is dominant over a significant, not fine-tuned, range of parameters of the NMSSM). The second exposes a limited portion of parameter space for which Higgs discovery would not have occurred at LEP and will probably not be possible at the LHC.

## Acknowledgments

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```

# INPUT FILE FOR NMHDECAY
# BASED ON SUSY LES HOUCHE ACCORD, MODIFIED FOR THE NMSSM
# IN EXTPAR: LINES 61-64: NMSSM YUKAWA COUPLINGS AND TRILIN. SOFT TERMS
BLOCK MODSEL
 3      1      # NMSSM PARTICLE CONTENT
BLOCK SMINPUTS
 1  137.036 # ALPHA_EM^-1(0)
 2  1.16639D-5  # GF
 3  0.12      # ALPHA_S(MZ)
 4  91.187    # MZ
 5  4.24      # MB(MB), RUNNING B QUARK MASS
 6  175.      # TOP QUARK POLE MASS
 7  1.7771    # MTAU
BLOCK MINPAR
 3  5.      # TANBETA
BLOCK EXTPAR
 1  5.D2    # M1
 2  1.D3    # M2
 3  3.D3    # M3
11  1.5D3   # ATOP
12  1.5D3   # ABOT
13  1.5D3   # ATAU
23  180.    # MU
31  1.D3    # LEFT SELELECTRON
32  1.D3    # LEFT SMUON
33  1.D3    # LEFT STAU
34  1.D3    # RIGHT SELELECTRON
35  1.D3    # RIGHT SMUON
36  1.D3    # RIGHT STAU
41  1.D3    # LEFT 1ST GEN. SQUARKS
42  1.D3    # LEFT 2ND GEN. SQUARKS
43  1.D3    # LEFT 3RD GEN. SQUARKS
44  1.D3    # RIGHT U-SQUARKS
45  1.D3    # RIGHT C-SQUARKS
46  1.D3    # RIGHT T-SQUARKS
47  1.D3    # RIGHT D-SQUARKS
48  1.D3    # RIGHT S-SQUARKS
49  1.D3    # RIGHT B-SQUARKS
61  .3D0    # LAMBDA
62  .3D0    # KAPPA
63  200.    # A_LAMBDA
64  0.0     # A_KAPPA

```

Table 1: Sample `slhainp.dat` file.

```

# NMHDECAY OUTPUT IN SLHA FORMAT
# Info about spectrum calculator
BLOCK SPINFO      # Program information
  1  NMHDECAY    # spectrum calculator
  2  1.1         # version number
# Input parameters
BLOCK MODSEL
  3  1           # NMSSM PARTICLE CONTENT
BLOCK SMINPUTS
  1  1.37036000E+02  # ALPHA_EM^-1
  2  1.16639000E-05  # GF
  3  1.20000000E-01  # ALPHA_S(MZ)
  4  9.11870000E+01  # MZ
  5  4.24000000E+00  # MB(MB)
  6  1.75000000E+02  # MTOP (POLE MASS)
  7  1.77710000E+00  # MTAU
# SMINPUTS Beyond SLHA:
# MW:      0.80419998E+02
# MS:      0.19000000E+00
# MC:      0.16100000E+01
# VUS:     0.22200000E+00
# VCB:     0.41000000E-01
# VUB:     0.36000000E-02
BLOCK MINPAR
  3  5.00000000E+00  # TANBETA
BLOCK EXTPAR
  1  5.00000000E+02  # M1
  2  1.00000000E+03  # M2
  3  3.00000000E+03  # M3
 11  1.50000000E+03  # ATOP
 12  1.50000000E+03  # ABOTTOM
 13  1.50000000E+03  # ATAU
 23  1.80000000E+02  # MU
 33  1.00000000E+03  # LEFT STAU
 36  1.00000000E+03  # RIGHT STAU
 43  1.00000000E+03  # LEFT 3RD GEN. SQUARKS
 46  1.00000000E+03  # RIGHT T-SQUARKS
 49  1.00000000E+03  # RIGHT B-SQUARKS
 61  3.00000000E-01  # LAMBDA
 62  3.00000000E-01  # KAPPA
 63  2.00000000E+02  # A_LAMBDA
 64  0.00000000E+00  # A_KAPPA
#
BLOCK MASS  # Mass spectrum
# PDG Code  mass          particle
  25  1.16801961E+02  # lightest neutral scalar
  35  3.56992533E+02  # second neutral scalar
  45  5.91271441E+02  # third neutral scalar
  36  4.90592823E+01  # lightest pseudoscalar
  46  5.87736637E+02  # second pseudoscalar
  37  5.90714771E+02  # charged Higgs
1000022  1.66456703E+02  # neutralino(1)
1000023  -1.85971845E+02  # neutralino(2)
1000025  3.67762262E+02  # neutralino(3)
1000035  5.04634098E+02  # neutralino(4)
1000045  1.00711878E+03  # neutralino(5)
1000024  1.76259627E+02  # chargino(1)
1000037  1.00710838E+03  # chargino(2)
1000005  8.21132887E+02  # ~b_1
1000006  6.24445725E+02  # ~t_1
2000005  8.24131926E+02  # ~b_2
2000006  1.00488215E+03  # ~t_2

```

```

# 3*3 Higgs mixing
BLOCK HIGMIX
 1 1 -1.27437514E-01 # S_(1,1)
 1 2 -3.16386555E-02 # S_(1,2)
 1 3 -9.91341856E-01 # S_(1,3)
 2 1 -2.01793548E-01 # S_(2,1)
 2 2 -9.77759539E-01 # S_(2,2)
 2 3 5.71458423E-02 # S_(2,3)
 3 1 -9.71101974E-01 # S_(3,1)
 3 2 2.07328914E-01 # S_(3,2)
 3 3 1.18218769E-01 # S_(3,3)

# 2*2 Pseudoscalar Higgs mixing
BLOCK AMIX
 1 1 -2.37325986E-02 # P'_-(1,1)
 1 2 -9.99718342E-01 # P'_-(1,2)
 2 1 9.99718342E-01 # P'_-(2,1)
 2 2 -2.37325986E-02 # P'_-(2,2)

# Gaugino-Higgsino mixing
BLOCK NMIX # 5*5 Neutralino Mixing Matrix
 1 1 1.05202671E-01 # N_(1,1)
 1 2 -7.87589631E-02 # N_(1,2)
 1 3 6.92565025E-01 # N_(1,3)
 1 4 -6.93980327E-01 # N_(1,4)
 1 5 1.46541512E-01 # N_(1,5)
 2 1 -3.48143940E-02 # N_(2,1)
 2 2 3.76738427E-02 # N_(2,2)
 2 3 7.01102969E-01 # N_(2,3)
 2 4 7.06807991E-01 # N_(2,4)
 2 5 7.90299289E-02 # N_(2,5)
 3 1 -2.45218320E-02 # N_(3,1)
 3 2 9.59570918E-03 # N_(3,2)
 3 3 -1.58479313E-01 # N_(3,3)
 3 4 4.52366830E-02 # N_(3,4)
 3 5 9.85973910E-01 # N_(3,5)
 4 1 9.93510538E-01 # N_(4,1)
 4 2 1.73884449E-02 # N_(4,2)
 4 3 -5.24484443E-02 # N_(4,3)
 4 4 9.87396900E-02 # N_(4,4)
 4 5 1.15796113E-02 # N_(4,5)
 5 1 -7.47306864E-03 # N_(5,1)
 5 2 9.95983570E-01 # N_(5,2)
 5 3 3.06884365E-02 # N_(5,3)
 5 4 -8.37728195E-02 # N_(5,4)
 5 5 -1.10280322E-03 # N_(5,5)

BLOCK UMIX # Chargino U Mixing Matrix
 1 1 4.31428408E-02 # U_(1,1)
 1 2 -9.99068914E-01 # U_(1,2)
 2 1 9.99068914E-01 # U_(2,1)
 2 2 4.31428408E-02 # U_(2,2)

BLOCK VMIX # Chargino V Mixing Matrix
 1 1 1.18343118E-01 # V_(1,1)
 1 2 -9.92972762E-01 # V_(1,2)
 2 1 9.92972762E-01 # V_(2,1)
 2 2 1.18343118E-01 # V_(2,2)

```

Table 2: Corresponding `spectr.dat` output file.

```

#
# Total number of points scanned
#
1000
#
# Output format 0=short 1=long (not recommended for big scannings)
#
0
#
# lambda
#
0.5
0.5
#
# kappa
#
-0.15
-0.15
#
# tan(beta)
#
3.5
3.5
#
# mu
#
200.
200.
#
# A_lambda
#
780.
780.
#
# A_kappa
#
150.0
250.0
#
# Remaining soft terms (no scan)
#
mQ3= 1.D3
mU3= 1.D3
mD3= 1.D3
mL3= 1.D3
mE3= 1.D3
AU3= 1.5D3
AD3= 1.5D3
AE3= 1.5D3
mQ= 1.D3
mU= 1.D3
mD= 1.D3
mL= 1.D3
mE= 1.D3
M1= 5.D2
M2= 1.D3
M3= 3.D3

```

Table 3: The `scaninp.dat` file for parameter scan #2.

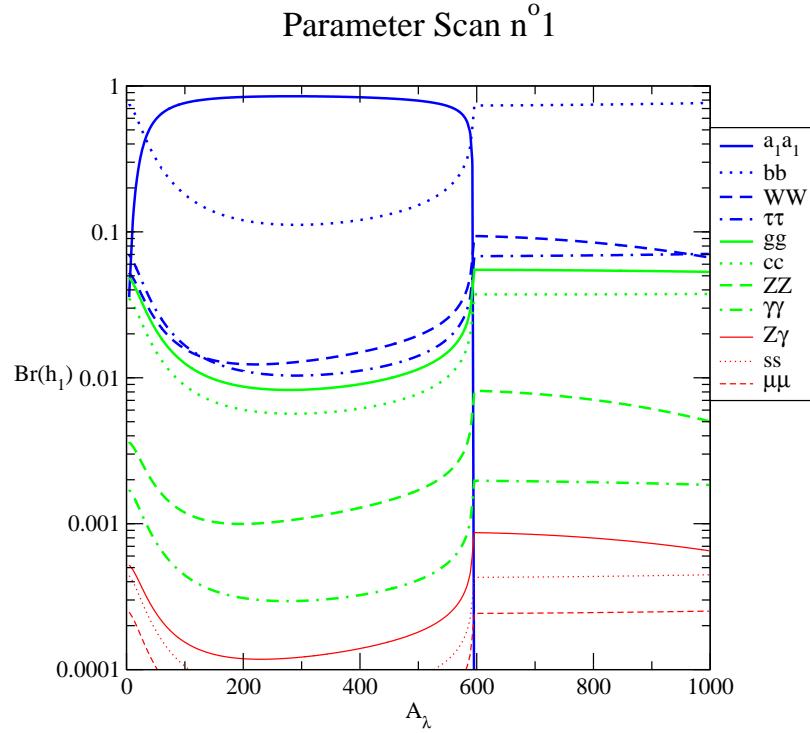


Figure 1: Branching ratios of  $h_1$  as a function of  $A_\lambda$  for  $\lambda = \kappa = 0.3$ ,  $\tan \beta = 5$ ,  $\mu_{\text{eff}} = 180$  GeV,  $A_\kappa = 0$ ,  $m_{\text{squark}} = 1$  TeV, and  $A_t = 1.5$  TeV.

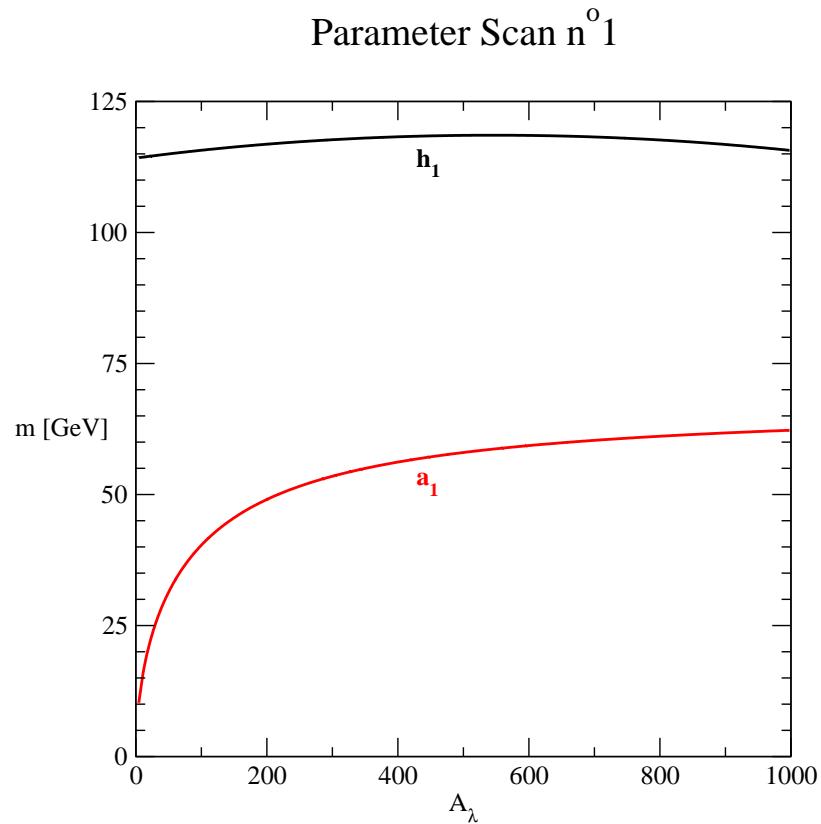


Figure 2:  $m_{h_1}$  and  $m_{a_1}$  as a function of  $A_\lambda$  for the same parameters as in fig. 1.

### Parameter Scan n<sup>o</sup>2

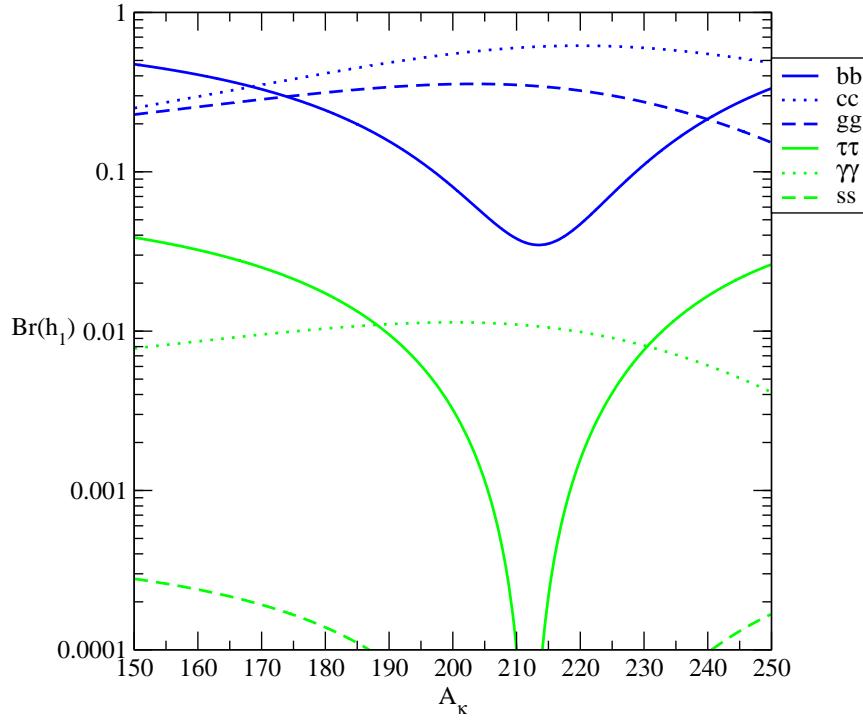


Figure 3: Branching ratios of  $h_1$  as a function of  $A_\kappa$  for  $\lambda = 0.5$ ,  $\kappa = -0.15$ ,  $\tan \beta = 3.5$ ,  $\mu_{\text{eff}} = 200$  GeV,  $A_\lambda = 780$  GeV,  $m_{\text{squark}} = 1$  TeV, and  $A_t = 1.5$  TeV.

### Parameter Scan n<sup>o</sup>2

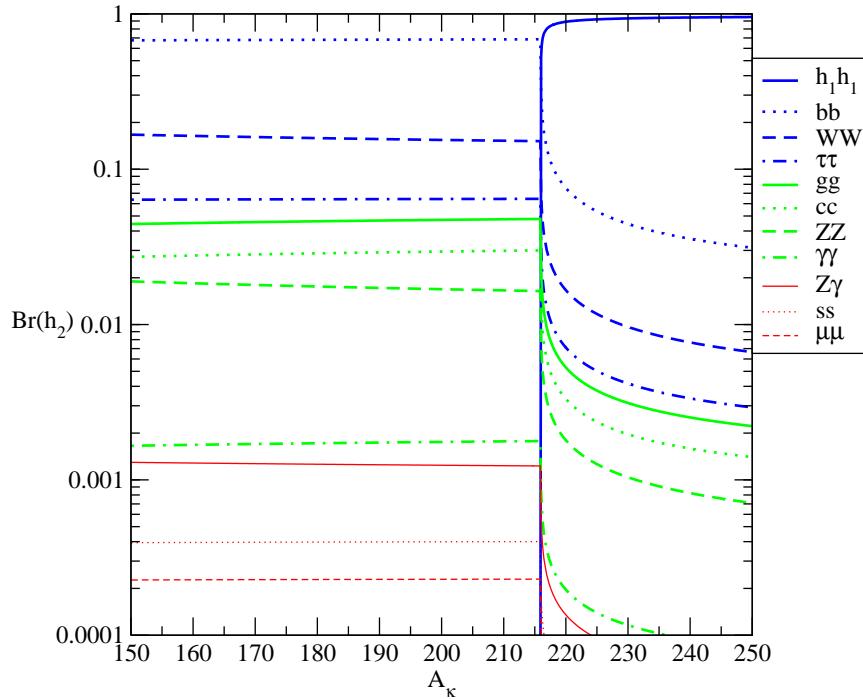


Figure 4: Branching ratios of  $h_2$  as a function of  $A_\kappa$  for the same parameter choices as in fig. 3.

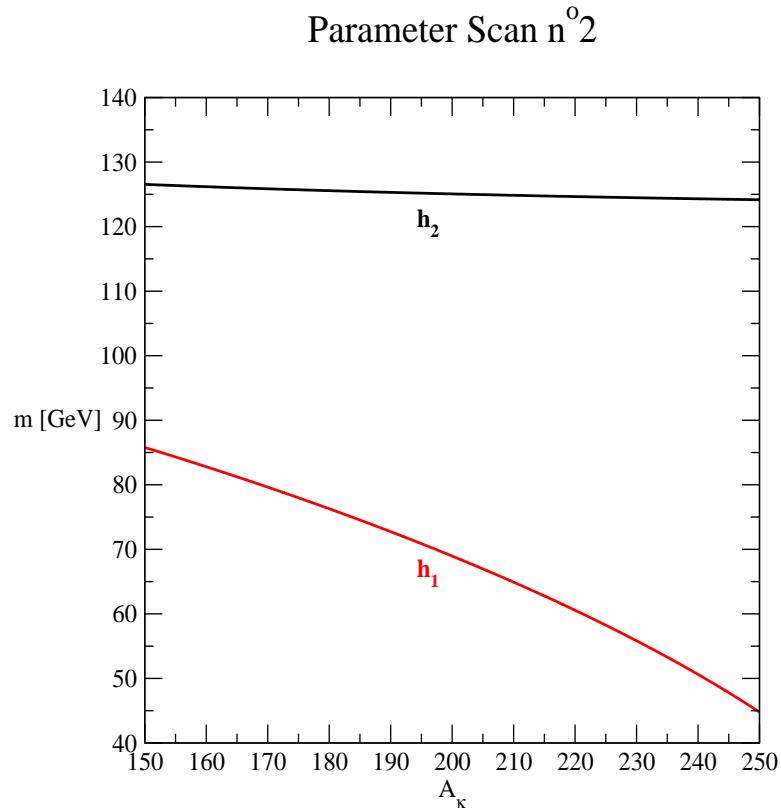


Figure 5:  $m_{h_1}$  and  $m_{h_2}$  as a function of  $A_\kappa$  for the same parameters as in fig. 3.

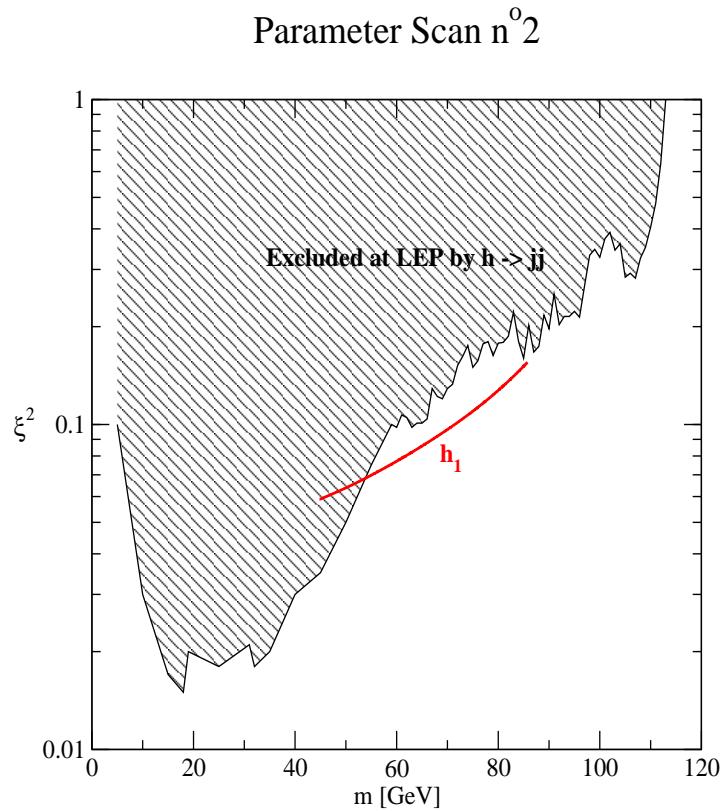


Figure 6: LEP constraints in comparison to predictions for  $h_1$  for the parameters of fig. 3. Note the correlation of  $m = m_{h_1}$  with  $A_\kappa$  given in fig. 5.

## Appendix A

### 1 General Conventions for the NMSSM

Below, we define our conventions for the tree level Lagrangian of the NMSSM. The superpotential for the Higgs fields, the quarks and the leptons of the 3rd generation is

$$W = h_t \hat{Q} \cdot \hat{H}_u \hat{T}_R^c - h_b \hat{Q} \cdot \hat{H}_d \hat{B}_R^c - h_\tau \hat{L} \cdot \hat{H}_d \hat{L}_R^c + \lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{1}{3} \kappa \hat{S}^3. \quad (\text{A.1})$$

Hereafter, hatted capital letters denote superfields, and unhatted capital letters the corresponding (complex) scalar components. The  $SU(2)$  doublets are

$$\hat{Q} = \begin{pmatrix} \hat{T}_L \\ \hat{B}_L \end{pmatrix}, \quad \hat{L} = \begin{pmatrix} \hat{\nu}_{\tau L} \\ \hat{\tau}_L \end{pmatrix}, \quad \hat{H}_u = \begin{pmatrix} \hat{H}_u^+ \\ \hat{H}_u^0 \end{pmatrix}, \quad \hat{H}_d = \begin{pmatrix} \hat{H}_d^0 \\ \hat{H}_d^- \end{pmatrix}. \quad (\text{A.2})$$

Products of two  $SU(2)$  doublets are defined as, e.g.,

$$\hat{H}_u \cdot \hat{H}_d = \hat{H}_u^+ \hat{H}_d^- - \hat{H}_u^0 \hat{H}_d^0. \quad (\text{A.3})$$

For the soft SUSY breaking terms we take

$$\begin{aligned} -\mathcal{L}_{\text{soft}} = & m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + m_Q^2 |Q^2| + m_T^2 |T_R^2| \\ & + m_B^2 |B_R^2| + m_L^2 |L^2| + m_\tau^2 |L_R^2| \\ & + (h_t A_t Q \cdot H_u T_R^c - h_b A_b Q \cdot H_d B_R^c - h_\tau A_\tau L \cdot H_d L_R^c \\ & + \lambda A_\lambda H_u \cdot H_d S + \frac{1}{3} \kappa A_\kappa S^3 + \text{h.c.}) \end{aligned} \quad (\text{A.4})$$

### 2 Higgs Sector at Tree Level

For completeness, we list here the Higgs potential, tree level Higgs masses and our conventions for the mixing angles. The tree level Higgs potential is given by

$$\begin{aligned} V = & \lambda^2 (|H_u|^2 |S|^2 + |H_d|^2 |S|^2 + |H_u \cdot H_d|^2) + \kappa^2 |S^2|^2 \\ & + \lambda \kappa (H_u \cdot H_d S^{*2} + \text{h.c.}) + \frac{1}{4} g^2 (|H_u|^2 - |H_d|^2)^2 \\ & + \frac{1}{2} g_2^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 \\ & + (\lambda A_\lambda H_u \cdot H_d S + \frac{1}{3} \kappa A_\kappa S^3 + \text{h.c.}) \end{aligned} \quad (\text{A.5})$$

where

$$g^2 = \frac{1}{2} (g_1^2 + g_2^2). \quad (\text{A.6})$$

Assuming vevs  $h_u$ ,  $h_d$  and  $s$  such that

$$H_u^0 = h_u + \frac{H_{uR} + iH_{uI}}{\sqrt{2}}, \quad H_d^0 = h_d + \frac{H_{dR} + iH_{dI}}{\sqrt{2}}, \quad S = s + \frac{S_R + iS_I}{\sqrt{2}} \quad (\text{A.7})$$

eq. (A.5) simplifies to

$$\begin{aligned} V = & \lambda^2 (h_u^2 s^2 + h_d^2 s^2 + h_u^2 h_d^2) + \kappa^2 s^4 - 2\lambda\kappa h_u h_d s^2 - 2\lambda A_\lambda h_u h_d s \\ & + \frac{2}{3} \kappa A_\kappa s^3 + m_{H_u}^2 h_u^2 + m_{H_d}^2 h_d^2 + m_S^2 s^2 + \frac{1}{4} g^2 (h_u^2 - h_d^2)^2. \end{aligned} \quad (\text{A.8})$$

The sign conventions for the fields can be chosen such that the Yukawa couplings  $\lambda$ ,  $h_t$ ,  $h_b$ , the vevs  $h_u$ ,  $h_d$  (and hence  $\tan\beta$ ) as well as the soft gaugino masses  $M_i$  are all positive. Then, the Yukawa coupling  $\kappa$ , the trilinear soft terms  $A_i$ , and the vev  $s$  (and hence  $\mu_{\text{eff}}$ ) can all be either positive or negative.

## 2.1 CP-even neutral states

In the basis  $S^{\text{bare}} = (H_{uR}, H_{dR}, S_R)$  and using the minimization equations in order to eliminate the soft masses squared, one obtains the following mass-squared matrix entries:

$$\begin{aligned} \mathcal{M}_{S,11}^2 &= g^2 h_u^2 + \lambda s \frac{h_d}{h_u} (A_\lambda + \kappa s), \\ \mathcal{M}_{S,22}^2 &= g^2 h_d^2 + \lambda s \frac{h_u}{h_d} (A_\lambda + \kappa s), \\ \mathcal{M}_{S,33}^2 &= \lambda A_\lambda \frac{h_u h_d}{s} + \kappa s (A_\kappa + 4\kappa s), \\ \mathcal{M}_{S,12}^2 &= (2\lambda^2 - g^2) h_u h_d - \lambda s (A_\lambda + \kappa s), \\ \mathcal{M}_{S,13}^2 &= 2\lambda^2 h_u s - \lambda h_d (A_\lambda + 2\kappa s), \\ \mathcal{M}_{S,23}^2 &= 2\lambda^2 h_d s - \lambda h_u (A_\lambda + 2\kappa s). \end{aligned} \quad (\text{A.9})$$

After diagonalization by an orthogonal matrix  $S_{ij}$  one obtains 3 CP-even states (ordered in mass)  $h_i = S_{ij} S_j^{\text{bare}}$ , with masses denoted by  $m_{h_i}$ .

In the MSSM limit ( $\lambda, \kappa \rightarrow 0$ , and parameters such that  $h_3 \sim S_R$ ) the elements of the first  $2 \times 2$  sub-matrix of  $S_{ij}$  are related to the MSSM angle  $\alpha$  as

$$\begin{aligned} S_{11} &\sim \cos \alpha, & S_{21} &\sim \sin \alpha, \\ S_{12} &\sim -\sin \alpha, & S_{22} &\sim \cos \alpha. \end{aligned} \quad (\text{A.10})$$

## 2.2 CP-odd neutral states

In the basis  $P^{bare} = (H_{uI}, H_{dI}, S_I)$  and using the minimization equations in order to eliminate the soft masses squared, one obtains the following mass-squared matrix entries:

$$\begin{aligned}
\mathcal{M}_{P,11}^2 &= \lambda s \frac{h_d}{h_u} (A_\lambda + \kappa s), \\
\mathcal{M}_{P,22}^2 &= \lambda s \frac{h_u}{h_d} (A_\lambda + \kappa s), \\
\mathcal{M}_{P,33}^2 &= 4\lambda\kappa h_u h_d + \lambda A_\lambda \frac{h_u h_d}{s} - 3\kappa A_\kappa s, \\
\mathcal{M}_{P,12}^2 &= \lambda s (A_\lambda + \kappa s), \\
\mathcal{M}_{P,13}^2 &= \lambda h_d (A_\lambda - 2\kappa s), \\
\mathcal{M}_{P,23}^2 &= \lambda h_u (A_\lambda - 2\kappa s).
\end{aligned} \tag{A.11}$$

The diagonalization of this mass matrix is performed in two steps. First, one rotates into a basis  $(\tilde{A}, \tilde{G}, S_I)$ , where  $\tilde{G}$  is a massless Goldstone mode:

$$\begin{pmatrix} H_{uI} \\ H_{dI} \\ S_I \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta & 0 \\ \sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{A} \\ \tilde{G} \\ S_I \end{pmatrix} \tag{A.12}$$

where  $\tan \beta = h_u/h_d$ . Dropping the Goldstone mode, the remaining  $2 \times 2$  mass matrix in the basis  $(\tilde{A}, S_I)$  has the matrix elements

$$\begin{aligned}
\mathcal{M}_{P,11}^2 &= \lambda s \frac{h_u^2 + h_d^2}{h_u h_d} (A_\lambda + \kappa s), \\
\mathcal{M}_{P,22}^2 &= 4\lambda\kappa h_u h_d + \lambda A_\lambda \frac{h_u h_d}{s} - 3\kappa A_\kappa s, \\
\mathcal{M}_{P,12}^2 &= \lambda \sqrt{h_u^2 + h_d^2} (A_\lambda - 2\kappa s).
\end{aligned} \tag{A.13}$$

It can be diagonalized by an orthogonal  $2 \times 2$  matrix  $P'_{ij}$  such that the physical CP-odd states  $a_i$  (ordered in mass) are

$$\begin{aligned}
a_1 &= P'_{11} \tilde{A} + P'_{12} S_I \\
&= P'_{11} (\cos \beta H_{uI} + \sin \beta H_{dI}) + P'_{12} S_I, \\
a_2 &= P'_{21} \tilde{A} + P'_{22} S_I \\
&= P'_{21} (\cos \beta H_{uI} + \sin \beta H_{dI}) + P'_{22} S_I,
\end{aligned} \tag{A.14}$$

and, for completeness,

$$\tilde{G} = -\sin \beta H_{uI} + \cos \beta H_{dI}. \tag{A.15}$$

The decomposition of the bare states in terms of physical states reads

$$\begin{aligned}
H_{uI} &= \cos \beta (P'_{11} a_1 + P'_{21} a_2) - \sin \beta \tilde{G} , \\
H_{dI} &= \sin \beta (P'_{11} a_1 + P'_{21} a_2) + \cos \beta \tilde{G} , \\
S_I &= P'_{12} a_1 + P'_{22} a_2 .
\end{aligned} \tag{A.16}$$

(In principle, since the matrix  $P'_{ij}$  is orthogonal, it could be parameterized by one angle.) Eqs. (A.16) suggest the introduction of a  $2 \times 3$  matrix  $P_{ij}$  with

$$P_{i1} = \cos \beta P'_{i1}, P_{i2} = \sin \beta P'_{i1}, P_{i3} = P'_{i2} \tag{A.17}$$

such that, omitting the Goldstone boson,

$$\begin{aligned}
H_{uI} &= P_{11} a_1 + P_{21} a_2 , \\
H_{dI} &= P_{11} a_1 + P_{21} a_2 , \\
S_I &= P_{13} a_1 + P_{23} a_2 .
\end{aligned} \tag{A.18}$$

## 2.3 Charged states

In the basis  $(H_u^+, [H_d^-]^* = H_d^+)$ , the charged Higgs mass matrix is given by

$$\mathcal{M}_\pm^2 = \left( \lambda s (A_\lambda + \kappa s) + h_u h_d \left( \frac{g_2^2}{2} - \lambda^2 \right) \right) \begin{pmatrix} \cot \beta & 1 \\ 1 & \tan \beta \end{pmatrix}. \tag{A.19}$$

This gives one eigenstate  $H^\pm$  of mass  $\text{Tr} \mathcal{M}_\pm^2$  and one massless goldstone mode  $G^\pm$  with

$$\begin{aligned}
H_u^\pm &= \cos \beta H^\pm - \sin \beta G^\pm , \\
H_d^\pm &= \sin \beta H^\pm + \cos \beta G^\pm .
\end{aligned} \tag{A.20}$$

## 3 SUSY Particles

### 3.1 Neutralinos

Denoting the  $U(1)_Y$  gaugino by  $\lambda_1$  and the neutral  $SU(2)$  gaugino by  $\lambda_2^3$ , the mass terms in the Lagrangian read

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2} M_1 \lambda_1 \lambda_1 + \frac{1}{2} M_2 \lambda_2^3 \lambda_2^3 \\
&\quad + \lambda (s \psi_u^0 \psi_d^0 + h_u \psi_d^0 \psi_s + h_d \psi_u^0 \psi_s) - \kappa s \psi_s \psi_s \\
&\quad + \frac{ig_1}{\sqrt{2}} \lambda_1 (h_u \psi_u^0 - h_d \psi_d^0) - \frac{ig_2}{\sqrt{2}} \lambda_2^3 (h_u \psi_u^0 - h_d \psi_d^0) .
\end{aligned} \tag{A.21}$$

In the basis  $\psi^0 = (-i\lambda_1, -i\lambda_2, \psi_u^0, \psi_d^0, \psi_s)$  one can rewrite

$$\mathcal{L} = -\frac{1}{2} (\psi^0)^T \mathcal{M}_0 (\psi^0) + \text{h.c.} \quad (\text{A.22})$$

where

$$\mathcal{M}_0 = \begin{pmatrix} M_1 & 0 & \frac{g_1 h_u}{\sqrt{2}} & -\frac{g_1 h_d}{\sqrt{2}} & 0 \\ 0 & M_2 & -\frac{g_2 h_u}{\sqrt{2}} & \frac{g_2 h_d}{\sqrt{2}} & 0 \\ 0 & 0 & -\mu & -\lambda h_d & 0 \\ 0 & 0 & 0 & -\lambda h_u & 2\kappa s \end{pmatrix}. \quad (\text{A.23})$$

(Recall that here  $\mu = \mu_{\text{eff}} = \lambda s$ ). One obtains 5 eigenstates (ordered in mass)  $\chi_i^0 = N_{ij} \psi_j^0$ , with  $N_{ij}$  real, with masses  $m_{\chi_i^0}$  that are real, but not necessarily positive.

### 3.2 Charginos

The charged  $SU(2)$  gauginos are  $\lambda^- = \frac{1}{\sqrt{2}} (\lambda_2^1 + i\lambda_2^2)$ ,  $\lambda^+ = \frac{1}{\sqrt{2}} (\lambda_2^1 - i\lambda_2^2)$ .

Defining

$$\psi^+ = \begin{pmatrix} -i\lambda^+ \\ \psi_u^+ \end{pmatrix}, \quad \psi^- = \begin{pmatrix} -i\lambda^- \\ \psi_d^- \end{pmatrix} \quad (\text{A.24})$$

the Lagrangian can be written as

$$\mathcal{L} = -\frac{1}{2} (\psi^+, \psi^-) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{h.c.} \quad (\text{A.25})$$

with

$$X = \begin{pmatrix} M_2 & g_2 h_u \\ g_2 h_d & \mu \end{pmatrix}. \quad (\text{A.26})$$

The mass eigenstates are  $\chi^+ = V\psi^+$ ,  $\chi^- = U\psi^-$ , with

$$U = \begin{pmatrix} \cos \theta_U & \sin \theta_U \\ -\sin \theta_U & \cos \theta_U \end{pmatrix}, \quad V = \begin{pmatrix} \cos \theta_V & \sin \theta_V \\ -\sin \theta_V & \cos \theta_V \end{pmatrix}. \quad (\text{A.27})$$

Defining

$$\gamma = \sqrt{\text{Tr}(X^T X) - 4\det(X^T X)} \quad (\text{A.28})$$

one has

$$\begin{aligned} \tan \theta_U &= \frac{g_2^2(h_d^2 - h_u^2) + \mu^2 - M_2^2 - \gamma}{2g_2(M_2 h_u + \mu h_d)}, \\ \tan \theta_V &= \frac{g_2^2(h_u^2 - h_d^2) + \mu^2 - M_2^2 - \gamma}{2g_2(M_2 h_d + \mu h_u)} \end{aligned} \quad (\text{A.29})$$

where  $-\pi/2 \leq \theta_U, \theta_V \leq \pi/2$  are such that  $M_D = UXV^T$  is diagonal, but not necessarily positive. The masses with  $|m_{\tilde{\chi}_1}| < |m_{\tilde{\chi}_2}|$  are given by

$$\begin{aligned} m_{\tilde{\chi}_1} &= \cos \theta_U (M_2 \cos \theta_V + g_2 h_u \sin \theta_V) + \sin \theta_U (g_2 h_d \cos \theta_V + \mu \sin \theta_V), \\ m_{\tilde{\chi}_2} &= \sin \theta_U (M_2 \sin \theta_V - g_2 h_u \cos \theta_V) - \cos \theta_U (g_2 h_d \sin \theta_V - \mu \cos \theta_V). \end{aligned} \quad (\text{A.30})$$

In terms of 4 component Dirac spinors  $\Psi_i = \begin{pmatrix} \chi_i^+ \\ \bar{\chi}_i^- \end{pmatrix}$  one can rewrite the Lagrangian as

$$\mathcal{L} = -\chi^- M_D \chi^+ + \text{h.c.} = -m_{\tilde{\chi}_1} \bar{\Psi}_1 \Psi_1 - m_{\tilde{\chi}_2} \bar{\Psi}_2 \Psi_2. \quad (\text{A.31})$$

### 3.3 Top and Bottom Squarks

To complete the consequences of our conventions above, we give here the top and bottom squark mass-squared matrices (without the D-term contributions). Below,  $t_L$ ,  $t_R^c$ ,  $b_L$  and  $b_R^c$  denote the two component quark spinors.

Top squarks:

$$\begin{pmatrix} T_R^* & T_L \\ T_L^* & \end{pmatrix} = \begin{pmatrix} T_R & h_t(A_t h_u - \lambda s h_d) \\ h_t(A_t h_u - \lambda s h_d) & m_Q^2 + h_t^2 h_u^2 \end{pmatrix} \quad (\text{A.32})$$

Bottom squarks:

$$\begin{pmatrix} B_R^* & B_L \\ B_L^* & \end{pmatrix} = \begin{pmatrix} B_R & h_b(A_b h_d - \lambda s h_u) \\ h_b(A_b h_d - \lambda s h_u) & m_Q^2 + h_b^2 h_d^2 \end{pmatrix} \quad (\text{A.33})$$

## Appendix B

# 1 Feynman rules for the Higgs Couplings

## 1.1 Higgs-Quarks

The couplings are obtained by expanding the quark mass matrices in the (properly normalized) physical Higgs fields  $h_i$ ,  $a_i$  and  $H^\pm$ . Below, we use  $v^2 = h_u^2 + h_d^2$ , and consider the quarks of the third generation.

$$\begin{aligned}
h_i t_L t_R^c &: \frac{m_t}{\sqrt{2}v \sin \beta} S_{i1} \\
h_i b_L b_R^c &: \frac{m_b}{\sqrt{2}v \cos \beta} S_{i2} \\
a_i t_L t_R^c &: i \frac{m_t}{\sqrt{2}v \sin \beta} P_{i1} \\
a_i b_L b_R^c &: i \frac{m_b}{\sqrt{2}v \cos \beta} P_{i2} \\
H^+ b_L t_R^c &: -\frac{m_t}{v} \cot \beta \\
H^- t_L b_R^c &: -\frac{m_b}{v} \tan \beta
\end{aligned} \tag{B.1}$$

## 1.2 Higgs-Gauge Bosons

These couplings are obtained from the kinetic terms in the Lagrangian:

$$\begin{aligned}
h_i Z_\mu Z_\nu &: g_{\mu\nu} \frac{g_1^2 + g_2^2}{\sqrt{2}} (h_u S_{i1} + h_d S_{i2}) \\
h_i W_\mu^+ W_\nu^- &: g_{\mu\nu} \frac{g_2^2}{\sqrt{2}} (h_u S_{i1} + h_d S_{i2}) \\
h_i(p) H^+(p') W_\mu^- &: \frac{g_2}{2} (\cos \beta S_{i1} - \sin \beta S_{i2}) (p - p')_\mu \\
a_i(p) H^+(p') W_\mu^- &: i \frac{g_2}{2} (\cos \beta P_{i1} + \sin \beta P_{i2}) (p - p')_\mu \\
h_i(p) a_j(p') Z_\mu &: i \frac{g}{\sqrt{2}} (S_{i1} P_{j1} - S_{i2} P_{j2}) (p - p')_\mu \\
H^+(p) H^-(p') Z_\mu &: \frac{g_1^2 - g_2^2}{\sqrt{g_1^2 + g_2^2}} (p - p')_\mu
\end{aligned} \tag{B.2}$$

## 1.3 Higgs-Neutralinos/Charginos

As in the case of the Higgs-Quark couplings, these couplings are obtained by expanding the corresponding mass matrices:

$$\begin{aligned}
h_a \chi_i^+ \chi_j^- &: \frac{\lambda}{\sqrt{2}} S_{a3} U_{i2} V_{j2} + \frac{g_2}{\sqrt{2}} (S_{a1} U_{i1} V_{j2} + S_{a2} U_{i2} V_{j1}) \\
a_a \chi_i^+ \chi_j^- &: i \left( \frac{\lambda}{\sqrt{2}} P_{a3} U_{i2} V_{j2} - \frac{g_2}{\sqrt{2}} (P_{a1} U_{i1} V_{j2} + P_{a2} U_{i2} V_{j1}) \right) \\
H^+ \chi_i^- \chi_j^0 &: \lambda \cos \beta U_{i2} N_{j5} - \frac{\sin \beta}{\sqrt{2}} U_{i2} (g_1 N_{j1} + g_2 N_{j2}) + g_2 \sin \beta U_{i1} N_{j4} \\
H^- \chi_i^+ \chi_j^0 &: \lambda \sin \beta V_{i2} N_{j5} + \frac{\cos \beta}{\sqrt{2}} V_{i2} (g_1 N_{j1} + g_2 N_{j2}) + g_2 \cos \beta V_{i1} N_{j3} \\
h_a \chi_i^0 \chi_j^0 &: \frac{\lambda}{\sqrt{2}} (S_{a1} \Pi_{ij}^{45} + S_{a2} \Pi_{ij}^{35} + S_{a3} \Pi_{ij}^{34}) - \sqrt{2} \kappa S_{a3} N_{i5} N_{j5} \\
&\quad - \frac{g_1}{2} (S_{a1} \Pi_{ij}^{13} - S_{a2} \Pi_{ij}^{14}) + \frac{g_2}{2} (S_{a1} \Pi_{ij}^{23} - S_{a2} \Pi_{ij}^{24}) \\
a_a \chi_i^0 \chi_j^0 &: i \left( \frac{\lambda}{\sqrt{2}} (P_{a1} \Pi_{ij}^{45} + P_{a2} \Pi_{ij}^{35} + P_{a3} \Pi_{ij}^{34}) - \sqrt{2} \kappa P_{a3} N_{i5} N_{j5} \right. \\
&\quad \left. + \frac{g_1}{2} (P_{a1} \Pi_{ij}^{13} - P_{a2} \Pi_{ij}^{14}) - \frac{g_2}{2} (P_{a1} \Pi_{ij}^{23} - P_{a2} \Pi_{ij}^{24}) \right) \tag{B.3}
\end{aligned}$$

where  $\Pi_{ij}^{ab} = N_{ia} N_{jb} + N_{ib} N_{ja}$ .

## 1.4 Triple Higgs Interactions

The trilinear Higgs self-couplings are obtained by expanding the scalar potential.

$$\begin{aligned}
h_a h_b h_c &: \frac{\lambda^2}{\sqrt{2}} (h_u (\Pi_{abc}^{122} + \Pi_{abc}^{133}) + h_d (\Pi_{abc}^{211} + \Pi_{abc}^{233}) + s (\Pi_{abc}^{311} + \Pi_{abc}^{322})) \\
&\quad - \frac{\lambda \kappa}{\sqrt{2}} (h_u \Pi_{abc}^{323} + h_d \Pi_{abc}^{313} + 2s \Pi_{abc}^{123}) + \sqrt{2} \kappa^2 s \Pi_{abc}^{333} \\
&\quad - \frac{\lambda A_\lambda}{\sqrt{2}} \Pi_{abc}^{123} + \frac{\kappa A_\kappa}{3\sqrt{2}} \Pi_{abc}^{333} \\
&\quad + \frac{g^2}{2\sqrt{2}} (h_u (\Pi_{abc}^{111} - \Pi_{abc}^{122}) - h_d (\Pi_{abc}^{211} - \Pi_{abc}^{222})) \tag{B.4}
\end{aligned}$$

where

$$\begin{aligned}
\Pi_{abc}^{ijk} &= S_{ai} S_{bj} S_{ck} + S_{ai} S_{cj} S_{bk} + S_{bi} S_{aj} S_{ck} \\
&\quad + S_{bi} S_{cj} S_{ak} + S_{ci} S_{aj} S_{bk} + S_{ci} S_{bj} S_{ak} . \tag{B.5}
\end{aligned}$$

$$\begin{aligned}
h_a a_b a_c &: \frac{\lambda^2}{\sqrt{2}} \left( h_u (\Pi_{abc}^{122} + \Pi_{abc}^{133}) + h_d (\Pi_{abc}^{211} + \Pi_{abc}^{233}) + s (\Pi_{abc}^{311} + \Pi_{abc}^{322}) \right) \\
&+ \frac{\lambda \kappa}{\sqrt{2}} \left( h_u (\Pi_{abc}^{233} - 2 \Pi_{abc}^{323}) + h_d (\Pi_{abc}^{133} - 2 \Pi_{abc}^{313}) \right. \\
&\left. + 2s (\Pi_{abc}^{312} - \Pi_{abc}^{123} - \Pi_{abc}^{213}) \right) + \sqrt{2} \kappa^2 s \Pi_{abc}^{333} \\
&+ \frac{\lambda A_\lambda}{\sqrt{2}} (\Pi_{abc}^{123} + \Pi_{abc}^{213} + \Pi_{abc}^{312}) - \frac{\kappa A_\kappa}{\sqrt{2}} \Pi_{abc}^{333} \\
&+ \frac{g^2}{2\sqrt{2}} \left( h_u (\Pi_{abc}^{111} - \Pi_{abc}^{122}) - h_d (\Pi_{abc}^{211} - \Pi_{abc}^{222}) \right)
\end{aligned} \tag{B.6}$$

where

$$\Pi_{abc}^{ijk} = S_{ai} (P_{bj} P_{ck} + P_{cj} P_{bk}) . \tag{B.7}$$

$$\begin{aligned}
h_a H^+ H^- &: \frac{\lambda^2}{\sqrt{2}} (s (\Pi_a^{311} + \Pi_a^{322}) - h_u \Pi_a^{212} - h_d \Pi_a^{112}) \\
&+ \sqrt{2} \lambda \kappa s \Pi_a^{312} + \frac{\lambda A_\lambda}{\sqrt{2}} \Pi_a^{312} \\
&+ \frac{g_1^2}{4\sqrt{2}} \left( h_u (\Pi_a^{111} - \Pi_a^{122}) + h_d (\Pi_a^{222} - \Pi_a^{211}) \right) \\
&+ \frac{g_2^2}{4\sqrt{2}} \left( h_u (\Pi_a^{111} + \Pi_a^{122} + 2 \Pi_a^{212}) + h_d (\Pi_a^{211} + \Pi_a^{222} + 2 \Pi_a^{112}) \right)
\end{aligned} \tag{B.8}$$

where

$$\Pi_a^{ijk} = 2 S_{ai} C_j C_k \tag{B.9}$$

with  $C_1 = \cos \beta$ ,  $C_2 = \sin \beta$ .

## Appendix C

In this appendix we give the explicit expressions of the radiative corrections to the Higgs masses.

Generally, the radiative corrections to the Higgs effective action consist in corrections to the wave function renormalization constants  $Z_i$ , and corrections to the effective potential  $V_{eff}(h_i)$ . We will treat the one loop corrections originating from stop and sbottom loops exactly in the stop and sbottom mixings and mass splittings. The resulting contributions to the Higgs masses are nevertheless quite simple if they are expressed in terms of couplings and masses at the scale  $Q = M_{SUSY}$  (the average of the squark masses, see below) and in terms of the Higgs vevs *before* rescaling by  $Z_i^{1/2}$ , which we will denote by  $h_i(Q)$ . However, the Higgs vevs are related to the Fermi coupling  $G_F$  and our definition of  $\tan \beta$  *after* rescaling by  $Z_i^{1/2}$ , cf. eq. (2.9).

Hence, we first have to determine  $Z_i$  for  $i = H_u, H_d$ :

$$\begin{aligned} Z_{H_u} &= 1 + \frac{3h_t^2}{16\pi^2}t \\ Z_{H_d} &= 1 + \frac{3h_b^2}{16\pi^2}t \end{aligned} \quad (C.1)$$

where

$$\begin{aligned} t &= \ln(Q^2/m_{top}^2) , \\ Q^2 &= \frac{1}{4}(2m_Q^2 + m_T^2 + m_B^2) + m_{top}^2 \end{aligned} \quad (C.2)$$

(such that  $t = 0$  for vanishing SUSY breaking squark masses  $m_Q$ ,  $m_T$  and  $m_B$ ).

Now we have

$$h_u(Q) = h_u/\sqrt{Z_{H_u}} , \quad h_d(Q) = h_d/\sqrt{Z_{H_d}} . \quad (C.3)$$

with  $h_u$  and  $h_d$  given in terms of  $G_F$  and  $\tan \beta$ .

In addition we need the running Yukawa couplings and quark masses at the scale  $Q$ . First, the running bottom quark mass (and hence  $h_b$ ) at the scale  $m_{top}$  is given by its QCD evolution to one loop order

$$h_b(m_{top}) = h_b(m_{bot}) \left( 1 - \frac{23}{12\pi} \alpha_s(m_{top}) \ln \frac{m_{top}^2}{m_{bot}^2} \right)^{12/23} . \quad (C.4)$$

The running top quark Yukawa coupling at the scale  $m_{top}$  is given in terms of the top quark pole mass as <sup>||</sup>

$$h_t(m_{top}) = \frac{m_{top}(\text{pole})}{h_u} \left( 1 + \frac{4\alpha_s(m_{top})}{3\pi} + \frac{11\alpha_s^2(m_{top})}{\pi^2} \right) . \quad (C.5)$$

---

<sup>||</sup>Here we use the  $\overline{MS}$  relation between the pole mass and the running mass. In the  $\overline{DR}$  scheme

Next, the running Yukawa couplings at the scale  $Q$  are obtained from the RG equations (for an effective 2 Higgs doublet model)

$$\begin{aligned}\frac{dh_t}{d \ln Q^2} &= \frac{h_t}{64\pi^2}(9h_t^2 + h_b^2) - h_t \frac{\alpha_s}{\pi} , \\ \frac{dh_b}{d \ln Q^2} &= \frac{h_b}{64\pi^2}(9h_b^2 + h_t^2) - h_b \frac{\alpha_s}{\pi} .\end{aligned}\quad (\text{C.6})$$

Its solutions, exact in  $\alpha_s$  but perturbative in the Yukawa couplings, are

$$\begin{aligned}h_t(Q) &= h_t(m_{top}) \left(1 + \frac{7}{4\pi}\alpha_s(m_{top})t\right)^{-4/7} \left(1 + \frac{1}{64\pi^2}(9h_t^2 + h_b^2)t\right) , \\ h_b(Q) &= h_b(m_{top}) \left(1 + \frac{7}{4\pi}\alpha_s(m_{top})t\right)^{-4/7} \left(1 + \frac{1}{64\pi^2}(9h_b^2 + h_t^2)t\right) .\end{aligned}\quad (\text{C.7})$$

However, possibly a  $SU(2)$  Higgs doublet has a mass much larger than  $m_{top}$ . Then it is practically degenerate, and its approximate masses are given in the NMSSM by

$$M_{HD} = \lambda s(A_\lambda + \kappa s)(\cot \beta + \tan \beta) . \quad (\text{C.8})$$

At scales below  $M_{HD}$  the RG equations read

$$\begin{aligned}\frac{dh_t}{d \ln Q^2} &= \frac{h_t}{64\pi^2}(9h_t^2 + 3h_b^2) - h_t \frac{\alpha_s}{\pi} , \\ \frac{dh_b}{d \ln Q^2} &= \frac{h_b}{64\pi^2}(9h_b^2 + 3h_t^2) - h_b \frac{\alpha_s}{\pi} .\end{aligned}\quad (\text{C.9})$$

In order to correct the expressions for  $h_t(Q)$ ,  $h_b(Q)$  in this case, we have to multiply them by  $\left(1 + \frac{h_b^2}{32\pi^2} \ln(M_{HD}^2/m_{top}^2)\right)$ ,  $\left(1 + \frac{h_t^2}{32\pi^2} \ln(M_{HD}^2/m_{top}^2)\right)$ , respectively, if  $M_{HD} > m_{top}$ .

The expressions for the running top and bottom quark masses at the scale  $Q$  are then simply

$$m_{top}(Q) = h_t(Q)h_u(Q) , \quad m_{bot}(Q) = h_b(Q)h_d(Q) , \quad (\text{C.10})$$

and below it is convenient to use

$$\tan \beta(Q) = h_u(Q)/h_d(Q) . \quad (\text{C.11})$$

For the (s)top, (s)bottom induced one loop corrections to  $V_{eff}$  we still need the eigenvalues of the stop, sbottom mass matrices (A.32) and (A.33). To this end it is the second factor  $4/3\pi$  would read  $5/3\pi$ , which would increase  $h_t(m_{top})$  and hence the lightest Higgs mass by  $\sim 1\%$ . Since we have not included subdominant (single) logarithms in the two loop corrections to the effective potential we are not sensitive to the scheme in which the running top quark mass is defined, which leads to a theoretical error of  $\sim 1\%$  on the mass of the lightest Higgs.

convenient to define, step by step,

$$\begin{aligned}
M_{stop}^2 &= \frac{1}{2}(m_Q^2 + m_T^2) , \\
M_{sbot}^2 &= \frac{1}{2}(m_Q^2 + m_B^2) , \\
\Delta M_{stop}^2 &= \frac{1}{2}(m_Q^2 - m_T^2) , \\
\Delta M_{sbot}^2 &= \frac{1}{2}(m_Q^2 - m_B^2) , \\
X_t &= A_t - \lambda s \cot \beta(Q) , \\
X_b &= A_b - \lambda s \tan \beta(Q) , \\
W_t &= \sqrt{\Delta M_{stop}^4 + m_{top}^2(Q) X_t^2} , \\
W_b &= \sqrt{\Delta M_{sbot}^4 + m_{bot}^2(Q) X_b^2} ,
\end{aligned} \tag{C.12}$$

with the help of which the eigenvalues of the stop, sbottom mass matrices become

$$\begin{aligned}
M_{stop1}^2 &= M_{stop}^2 + m_{top}^2(Q) - W_t , \\
M_{stop2}^2 &= M_{stop}^2 + m_{top}^2(Q) + W_t , \\
M_{sbot1}^2 &= M_{sbot}^2 + m_{bot}^2(Q) - W_b , \\
M_{sbot2}^2 &= M_{sbot}^2 + m_{bot}^2(Q) + W_b .
\end{aligned} \tag{C.13}$$

Finally the following auxiliary quantities are useful:

$$\begin{aligned}
fmt &= \frac{1}{2W_t} \left( M_{stop2}^2 \ln \left( \frac{M_{stop2}^2}{Q^2} \right) - M_{stop1}^2 \ln \left( \frac{M_{stop1}^2}{Q^2} \right) \right) - 1 , \\
gmt &= \frac{m_{top}^2 X_t^2}{W_t^2} \left( \frac{M_{stop2}^2 + M_{stop1}^2}{M_{stop2}^2 - M_{stop1}^2} \ln \left( \frac{M_{stop2}^2}{M_{stop1}^2} \right) - 2 \right) , \\
emt &= \frac{m_{top}^2 X_t}{W_t} \ln \left( \frac{M_{stop2}^2}{M_{stop1}^2} \right) , \\
fmb &= \frac{1}{2W_b} \left( M_{sbot2}^2 \ln \left( \frac{M_{sbot2}^2}{Q^2} \right) - M_{sbot1}^2 \ln \left( \frac{M_{sbot1}^2}{Q^2} \right) \right) - 1 , \\
gmb &= \frac{m_{bot}^2 X_b^2}{W_b^2} \left( \frac{M_{sbot2}^2 + M_{sbot1}^2}{M_{sbot2}^2 - M_{sbot1}^2} \ln \left( \frac{M_{sbot2}^2}{M_{sbot1}^2} \right) - 2 \right) , \\
emb &= \frac{m_{bot}^2 X_b}{W_b} \ln \left( \frac{M_{sbot2}^2}{M_{sbot1}^2} \right) ,
\end{aligned} \tag{C.14}$$

and, remarkably, quite many of the radiative corrections can be absorbed into a shift of  $A_\lambda$ :

$$A_{\lambda,new} = A_\lambda + \frac{3h_t(Q)^2}{16\pi^2} A_t \ fmt + \frac{3h_b(Q)^2}{16\pi^2} A_b \ fmb . \tag{C.15}$$

Subsequently the index *new* of  $A_{\lambda,new}$  will be omitted.

Then the radiatively corrected mass-squared matrix entries in the CP even sector are, instead of (A.9),

$$\begin{aligned}
\mathcal{M}_{S,11}^2 &= g^2 h_u(Q)^2 + \lambda s (A_\lambda + \kappa s) \cot \beta(Q) - \frac{3h_b(Q)^2}{32\pi^2} (\lambda s)^2 gmb \\
&\quad + \frac{3h_t(Q)^2}{32\pi^2} \left( -A_t^2 gmt + 4A_t emt + 4m_{top}^2 \ln \left( \frac{M_{stop1}^2 M_{stop2}^2}{m_{top}^4} \right) \right) , \\
\mathcal{M}_{S,22}^2 &= g^2 h_d(Q)^2 + \lambda s (A_\lambda + \kappa s) \tan \beta(Q) - \frac{3h_t(Q)^2}{32\pi^2} (\lambda s)^2 gmt \\
&\quad + \frac{3h_b(Q)^2}{32\pi^2} \left( -A_b^2 gmt + 4A_b emb + 4m_{bot}^2 \ln \left( \frac{M_{sbot1}^2 M_{sbot2}^2}{m_{bot}^4} \right) \right) , \\
\mathcal{M}_{S,33}^2 &= \lambda A_\lambda \frac{h_u(Q)h_d(Q)}{s} + \kappa s (A_\kappa + 4\kappa s) \\
&\quad - \frac{3h_t(Q)^2}{32\pi^2} \lambda^2 h_d(Q)^2 gmt - \frac{3h_b(Q)^2}{32\pi^2} \lambda^2 h_u(Q)^2 gmb , \\
\mathcal{M}_{S,12}^2 &= (2\lambda^2 - g^2) h_u(Q)h_d(Q) - \lambda s (A_\lambda + \kappa s) \\
&\quad + \lambda s \left( \frac{3h_t(Q)^2}{32\pi^2} (A_t gmt - 2 emt) + \frac{3h_b(Q)^2}{32\pi^2} (A_b gmb - 2 emb) \right) , \\
\mathcal{M}_{S,13}^2 &= 2\lambda^2 h_u(Q)s - \lambda h_d(Q)(A_\lambda + 2\kappa s) \\
&\quad + \frac{3h_t(Q)^2}{32\pi^2} \lambda h_d(Q)(A_t gmt - 2 emt) \\
&\quad + \frac{3h_b(Q)^2}{32\pi^2} \lambda^2 s h_u(Q)(4 fmb - gmb) , \\
\mathcal{M}_{S,23}^2 &= 2\lambda^2 h_d(Q)s - \lambda h_u(Q)(A_\lambda + 2\kappa s) \\
&\quad + \frac{3h_b(Q)^2}{32\pi^2} \lambda h_u(Q)(A_b gmb - 2 emb) \\
&\quad + \frac{3h_t(Q)^2}{32\pi^2} \lambda^2 s h_d(Q)(4 fmt - gmt) . \tag{C.16}
\end{aligned}$$

The dominant two loop effects  $\sim h_t^6 t^2$  and  $\sim h_t^4 \alpha_s t^2$  contribute to  $\mathcal{M}_{S,11}^2$  only. They read

$$\delta \mathcal{M}_{S,11 \text{ 2loops}}^2 = \frac{3h_t(Q)^4}{64\pi^4} h_u(Q)^2 \left( 32\pi\alpha_s(Q) - \frac{3}{2} h_t(Q)^2 \right) t^2 . \tag{C.17}$$

The above expressions for the elements of the CP even mass matrix have been obtained by taking the second derivatives of the effective potential, and using the three minimization equations with respect to  $h_u$ ,  $h_d$  and  $s$  in order to eliminate the three SUSY breaking masses squared. Since the Higgs fields are not quite properly normalized due to the  $Z$  factors, the above mass matrix elements still have to be rescaled by appropriate powers of  $Z_i$  (here below  $\mathcal{M}_{S,ij \text{ norm.}}^2$  denote the mass matrix

elements for correctly normalized Higgs fields):

$$\begin{aligned}
\mathcal{M}_{S,11 \text{ norm.}}^2 &= \mathcal{M}_{S,11}^2 / Z_{H_u} , \\
\mathcal{M}_{S,22 \text{ norm.}}^2 &= \mathcal{M}_{S,22}^2 / Z_{H_d} , \\
\mathcal{M}_{S,12 \text{ norm.}}^2 &= \mathcal{M}_{S,12}^2 / \sqrt{Z_{H_u} Z_{H_d}} , \\
\mathcal{M}_{S,13 \text{ norm.}}^2 &= \mathcal{M}_{S,13}^2 / \sqrt{Z_{H_u}} , \\
\mathcal{M}_{S,23 \text{ norm.}}^2 &= \mathcal{M}_{S,23}^2 / \sqrt{Z_{H_d}} .
\end{aligned} \tag{C.18}$$

Now we turn to the electroweak leading logarithmic corrections to the Higgs mass matrix elements, which are taken from ref. [15]. As in [15] we allow for a Higgs doublet which is possibly heavy ( $M_{HD} > M_Z$ ), and for gauginos/Higgsinos with masses possibly (much) larger than  $M_Z$ . In this case their masses are given approximately by the soft SUSY breaking gaugino mass parameters  $M_1$ ,  $M_2$  or  $\mu_{eff} = \lambda s$ , respectively. The following auxiliary parameters are useful subsequently:

$$\begin{aligned}
\mu_1 &= \max\{|\mu_{eff}|, M_1\} , \\
\mu_2 &= \max\{|\mu_{eff}|, M_2\} , \\
\mu_{12} &= \max\{|\mu_{eff}|, M_1, M_2\} .
\end{aligned} \tag{C.19}$$

The electroweak corrections to the Higgs mass matrix elements are then given by (we omit the index *norm.* in the following)

$$\begin{aligned}
\delta_{ew}\mathcal{M}_{S,11}^2 &= \frac{g_2^2 M_Z^4}{16\pi^2 M_W^2} \sin^2 \beta \left( (4 - 8 \sin^2 \theta_W + \frac{32}{3} \sin^4 \theta_W) t \right. \\
&\quad + \frac{1}{6} [-9(\cos^2 \beta - \sin^2 \beta)^4 + (1 - 2 \sin^2 \theta_W + 2 \sin^4 \theta_W)(\cos^2 \beta - \sin^2 \beta)^2 \\
&\quad + 10 - 2 \sin^2 \theta_W + 2 \sin^4 \theta_W] \ln(M_{HD}^2/M_Z^2) \\
&\quad + \sin^2 \theta_W (1 - 2 \sin^2 \theta_W) \ln(\mu_1^2/M_Z^2) - 4 \sin^2 \theta_W \cos^2 \theta_W \ln(\mu_{12}^2/M_Z^2) \\
&\quad + \cos^2 \theta_W (3 - 10 \cos^2 \theta_W) \ln(\mu_2^2/M_Z^2) - \frac{4}{3} \cos^4 \theta_W \ln(M_2^2/M_Z^2) \\
&\quad \left. - \frac{2}{3} (\sin^2 \theta_W + \cos^4 \theta_W) \ln(\mu_{eff}^2/M_Z^2) \right) , \\
\delta_{ew}\mathcal{M}_{S,22}^2 &= \frac{g_2^2 M_Z^4}{16\pi^2 M_W^2} \cos^2 \beta \left( (4 - 8 \sin^2 \theta_W + \frac{32}{3} \sin^4 \theta_W) t \right. \\
&\quad + \frac{1}{6} [-9(\cos^2 \beta - \sin^2 \beta)^4 + (1 - 2 \sin^2 \theta_W + 2 \sin^4 \theta_W)(\cos^2 \beta - \sin^2 \beta)^2 \\
&\quad + 10 - 2 \sin^2 \theta_W + 2 \sin^4 \theta_W] \ln(M_{HD}^2/M_Z^2) \\
&\quad + \sin^2 \theta_W (1 - 2 \sin^2 \theta_W) \ln(\mu_1^2/M_Z^2) - 4 \sin^2 \theta_W \cos^2 \theta_W \ln(\mu_{12}^2/M_Z^2) \\
&\quad + \cos^2 \theta_W (3 - 10 \cos^2 \theta_W) \ln(\mu_2^2/M_Z^2) - \frac{4}{3} \cos^4 \theta_W \ln(M_2^2/M_Z^2) \\
&\quad \left. - \frac{2}{3} (\sin^2 \theta_W + \cos^4 \theta_W) \ln(\mu_{eff}^2/M_Z^2) \right) , \\
\delta_{ew}\mathcal{M}_{S,12}^2 &= \frac{g_2^2 M_Z^4}{16\pi^2 M_W^2} \sin \beta \cos \beta \left( (-4 + 8 \sin^2 \theta_W - \frac{32}{3} \sin^4 \theta_W) t \right. \\
&\quad - \frac{1}{6} [-9(\cos^2 \beta - \sin^2 \beta)^4 + (1 - 2 \sin^2 \theta_W + 2 \sin^4 \theta_W)(\cos^2 \beta - \sin^2 \beta)^2 \\
&\quad + 8 - 22 \sin^2 \theta_W + 10 \sin^4 \theta_W] \ln(M_{HD}^2/M_Z^2) \\
&\quad - \sin^2 \theta_W (1 + 2 \sin^2 \theta_W) \ln(\mu_1^2/M_Z^2) - 4 \sin^2 \theta_W \cos^2 \theta_W \ln(\mu_{12}^2/M_Z^2) \\
&\quad - \cos^2 \theta_W (3 + 2 \cos^2 \theta_W) \ln(\mu_2^2/M_Z^2) + \frac{4}{3} \cos^4 \theta_W \ln(M_2^2/M_Z^2) \\
&\quad \left. + \frac{2}{3} (\sin^2 \theta_W + \cos^4 \theta_W) \ln(\mu_{eff}^2/M_Z^2) \right) . \tag{C.20}
\end{aligned}$$

The eigenvalues of this Higgs mass matrix give the "running" Higgs masses at a scale  $m_{top}$  due to the argument of the logarithm  $t$  in the  $Z$  factors (C.1). In the LLA we can identify the scales  $m_{top}$ ,  $M_Z$  (in the arguments of the logarithms) and, to start with,  $M_{Higgs}$ , and take these "running" masses as pole masses. However, if a Higgs mass is (much) larger than  $m_{top}$ , we should have computed the  $Z$  factors (C.1) at a scale  $M_{Higgs}$  rather than  $m_{top}$ , in order to identify the eigenvalues of the Higgs mass matrix with the Higgs pole masses. We can correct for this at the end (after having diagonalized the Higgs mass matrix), taking care that now the eigenstates  $h_i$  are given by  $h_i = S_{ij} S_j^{bare}$ , cf. the notation below eq. (A.9). Hence, if

$M_{Higgs} > m_{top}$ , the Higgs pole masses  $M_i^2(pole)$  are given in terms of the eigenvalues  $M_i^2$  as

$$M_i^2(pole) = M_i^2 / \left( 1 - \left( \frac{3h_t^2}{16\pi^2} S_{i1}^2 + \frac{3h_b^2}{16\pi^2} S_{i2}^2 \right) \ln(M_i^2/m_{top}^2) \right) . \quad (\text{C.21})$$

Now we turn to the radiative corrections to the CP odd mass matrix. After the shift (C.15) in  $A_\lambda$  these are trivial: It suffices to take the tree level expressions (A.11) for the mass matrix elements, to replace the Higgs vevs  $h_u$  and  $h_d$  by its expressions (C.3) at the scale  $Q$ , and to rescale the mass matrix elements by appropriate powers of  $Z_i$  as in (C.18) for the CP even sector. After dropping the Goldstone mode this gives a  $2 \times 2$  mass matrix (instead of (A.13))

$$\begin{aligned} \mathcal{M}_{P,11}^2 &= \lambda s(A_\lambda + \kappa s) \left( \frac{\tan \beta(Q)}{Z_{H_d}} + \frac{\cot \beta(Q)}{Z_{H_u}} \right) , \\ \mathcal{M}_{P,22}^2 &= 4\lambda\kappa h_u(Q)h_d(Q) + \lambda A_\lambda \frac{h_u(Q)h_d(Q)}{s} - 3\kappa A_\kappa s , \\ \mathcal{M}_{P,12}^2 &= \lambda \sqrt{h_u(Q)^2/Z_{H_d} + h_d(Q)^2/Z_{H_u}} (A_\lambda - 2\kappa s) . \end{aligned} \quad (\text{C.22})$$

Finally, if a CP odd Higgs mass is (much) larger than  $m_{top}$ , its pole mass is given by an expression analogous to (C.21) after replacing  $S_{i1}$  by  $P'_{i1} \cos^2 \beta$  and  $S_{i2}$  by  $P'_{i1} \sin^2 \beta$  (see (A.14) for the definition of  $P'_{ij}$ ).

In the expression for radiatively corrected charged Higgs mass we neglect stop/- sbottom loops (without large logarithms), but we include large logarithms proportional to electroweak gauge couplings:

$$\begin{aligned} M_{H^\pm}^2 &= (\lambda s(A_\lambda + \kappa s) + h_u(Q)h_d(Q)(g_2^2/2 - \lambda^2)) \left( \frac{\tan \beta(Q)}{Z_{H_d}} + \frac{\cot \beta(Q)}{Z_{H_u}} \right) \\ &\quad + 3h_t^2 h_b^2 t / (16\sqrt{2}\pi^2 G_F) \\ &\quad + \frac{g_2^2 M_W^2}{48\pi^2} (12t + 3 \tan^2 \theta_W (\ln(\mu_1^2/M_Z^2) + 4 \ln(\mu_{12}^2/M_Z^2)) \\ &\quad - 3 \ln(\mu_2^2/M_Z^2) - 4 \ln(M_2^2/M_Z^2) - 2 \ln(\mu_{eff}^2/M_Z^2)) \end{aligned} \quad (\text{C.23})$$

This time, once  $M_{H^\pm}$  is larger than  $m_{top}$ , its pole mass is given by

$$M_{H^\pm}^2(pole) = M_{H^\pm}^2 / \left( 1 - \left( \frac{3h_t^2}{16\pi^2} \cos^2 \beta + \frac{3h_b^2}{16\pi^2} \sin^2 \beta \right) \ln(M_{H^\pm}^2/m_{top}^2) \right) . \quad (\text{C.24})$$

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